Sec 2 Math: Simultaneous Equations

A) Simultaneous Equations

We will be expected to find the values of x and y in a given pair of equations. There will be three main methods of solving the equations:

1) Elimination
2) Substitution
3) Graphical Method

B) Solving by Elimination (Basic)

Solve the following pairs of simultaneous equations using the elimination method

$\boldsymbol{2x - 3y = 18}$
$\boldsymbol{3x + 2y = 1}$

Step 1: write and label the 2 equations:

$2x - 3y = 18$ --- Eqn 1
$3x + 2y = 1$ --- Eqn 2

Step 2: multiply each equation until the coefficients of one of the unknowns (x or y) is the same:

(Eqn 1) × 3: $6x - 9y = 54$ --- Eqn 3
(Eqn 2) × 2: $6x + 4y = 2$ --- Eqn 4

Step 3: Eliminate x by subtracting one Eqn by another:

(Eqn 3) – (Eqn 4):
$(6x - 9y) - (6x + 4y) = 54 - 2$
$-13y = 52$
$y = \frac{52}{-13} = -4$

Step 4: Find the other unknown by substituting the first answer into any of the equations:

Sub $y = -4$ into Eqn 1:
$2x - 3(-4) = 18$
$2x = 6$
$x = 3$
$\therefore x = 3, y = -4$

C) Solving by Substitution (Basic)

Solve the following pairs of simultaneous equations using the substitution method

$\boldsymbol{-2x + 9y = 8}$
$\boldsymbol{4x + 7y = 9}$

Step 1: write and label the 2 equations:

$-2x + 9y = 8$ --- Eqn 1
$4x + 7y = 9$ --- Eqn 2

Step 2: make one of the unknowns (x or y) the subject of one of the equations.

From Eqn 1:
$-2x + 9y = 8$
$2x = 9y - 8$
$x = \frac{9y-8}{2}$ --- Eqn 3

Step 3: Substitute Equation 3 into Equation 2.

$4x + 7y = 9$
$4\left(\frac{9y-8}{2}\right) + 7y = 9$
$4(9y - 8) + 14y = 18$
$36y - 32 + 14y - 18 = 0$
$50y = 50$
$y = 1$

Step 4: Find the other unknown by substituting first answer into any of the equations:

Sub $y = 1$ into Eqn 3:
$x = \frac{9(1)-8}{2}$
$x = 0.5$
$\therefore x = 0.5, y = 1$

D) Solving Simultaneous Equations Example (Intermediate/Advanced)

Solve the following pairs of simultaneous equations.

$\frac{5}{x} - \frac{2}{y} = 16$ **--- Eqn 1**
$\frac{2}{x} + \frac{3}{y} = 14$ **--- Eqn 2**

Note: For the above kind of question, it would be easier to solve by elimination

(Eqn 1) × 2 – (Eqn 2) × 5:
$\left(\frac{10}{x} - \frac{4}{y}\right) - \left(\frac{10}{x} + \frac{15}{y}\right) = 16 \times 2 - 14 \times 5$
$-\frac{19}{y} = -38$
$38y = 19$ Cross-Multiply
$y = \frac{19}{38} = 0.5$

Sub $y = 0.5$ into Eqn 1:
$\frac{5}{x} - \frac{2}{0.5} = 16$
$\frac{5}{x} = 20$
$20x = 5$ Cross-Multiply
$x = \frac{5}{20} = 0.25$
$\therefore x = 0.25, y = 0.5$

E) Simple word problem Example (Basic)

Mr McDonald has some chickens and cows in his farm. If he counted a total of 94 legs and 30 heads, find the number of chickens and the number of cows in his farm.

Let the number of chickens be x and the number of cows be y.

Chickens have 2 legs and cows have 4
$2x + 4y = 94$ --- Eqn 1
Each chicken and each cow has 1 head.
$x + y = 30$ --- Eqn 2

Solve Eqn 1 and Eqn 2 using either substitution of elimination… … …
$x = 13, y = 17$
There are 13 chickens and 17 cows.

F) Solving Single-line Simultaneous Equations Example (Intermediate)

Solve the following pairs of simultaneous equations

$\frac{1}{2}(x + 3y) = 3(2y - 3x - 1) = x + y + 2$

Note: Firstly, break the equation into 2

$\frac{(x+3y)}{2} = 3(2y - 3x - 1)$ --- Eqn 1
$3(2y - 3x - 1) = x + y + 2$ --- Eqn 2

From Eqn 1: $\frac{(x+3y)}{2} = 3(2y - 3x - 1)$
$(x + 3y) = 6(2y - 3x - 1)$
$x + 3y = 12y - 18x - 6$
$9y - 19x - 6 = 0$ --- Eqn 3

From Eqn 2: $3(2y - 3x - 1) = x + y + 2$
$6y - 9x - 3 = x + y + 2$
$5y - 10x - 5 = 0$
$y - 2x - 1 = 0$
$y = 2x + 1$ --- Eqn 4

Sub Eqn 4 into Eqn 3:
$9(2x + 1) - 19x - 6 = 0$
$18x + 9 - 19x - 6 = 0$
$x = 3$
Sub $x = 3$ into Eqn 4:
$y = 2(3) + 1 = 7$
$\therefore x = 3, y = 7$

G) Given Solutions find Unknowns (Example) (Intermediate)

If $x = 3$ and $y = -1$ are the solutions to the simultaneous equations:

$\boldsymbol{ax - by = 11}$
$\boldsymbol{3x + by = 5a}$

Find the values of a and b.

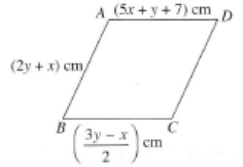
Since $x = 3$ and $y = -1$ are the solutions, substitute the values into the equations:

$a(3) - b(-1) = 11$ --- Eqn 1
$3(3) + b(-1) = 5a$ --- Eqn 2

From Eqn 1:
$3a + b = 11$
$b = 11 - 3a$ --- Eqn 3

Sub Eqn 3 into Eqn 2:
$3(3) + (11 - 3a)(-1) = 5a$
$9 - 11 + 3a - 5a = 0$
$a = -1$
Sub $a = -1$ in Eqn 3:
$b = 11 - 3(-1)$
$b = 14$
$\therefore a = -1, b = 14$

H) Simultaneous on Figures (Example) (Intermediate)



$ABCD$ is a rhombus.

Since all sides of a rhombus are equal:

$2y + x = 5x + y + 7$ --- Eqn 1
$2y + x = \frac{3y-x}{2}$ --- Eqn 2

Solve using either substitution or elimination …
…
$x = -1, y = 3$

I) ** Money Example (Intermediate)

Sandra paid for a dress that costs $125 using five-dollar and ten-dollar notes. She used 16 notes altogether. Using algebraic method, find the number of five-dollar notes she used.

Let the **number of** five-dollar notes be x.
Let the **number of** ten-dollar notes be y.

Form equation using "**number** of notes":
$x + y = 16$ ---Eqn 1

Form equation using "**total value** of money"
$5x + 10y = 125$ --- Eqn 2

Solve Eqn 1 and Eqn 2 using either substitution of elimination… … …
$x = 7,\ y = 9$
There are 7 five-dollar notes.

Note: Make sure to understand the difference between "number of notes" and "total value of money"

J) Two-Digit Number (Example 1) (*Advanced, **Important)

A two digit number is 4 times the sum of its digits. If the digits are reversed, the number will be increased by 27.
Find the number.

Let the tens digit of the original number be x and the ones digit be y.
$\therefore$ the original number is $10x + y$.
Key note: Please note that the two digit number is $10x + y$, NOT xy!!

$10x + y = 4(x + y)$ --- Eqn 1
$(10y + x) = (10x + y) + 27$ --- Eqn 2

Expand, simplify and Solve simultaneous:
$x = 3$ and $y = 6$
Therefore, the number is 36

K) Two-Digit Number (Example 2) (*Advanced, **Challenging)

A two digit number is such that the sum of its digits is one-seventh of the number. If the digits are reversed, the number will be decreased by 36. What is the number?

Let the tens digit of the original number be x and the ones digit be y.
$x + y = \frac{1}{7}(10x + y)$ --- Eqn 1
$(10x + y) - (10y + x) = 36$ --- Eqn 2

From Eqn 1:
$x + y = \frac{(10x+y)}{7}$
$7x + 7y = 10x + y$
$-3x + 6y = 0$ --- Eqn 3
From Eqn 2:
$10x + y - 10y - x = 36$
$9x - 9y = 36$ --- Eqn 4

Solve Eqn 3 and Eqn 4 using either substitution of elimination … …
$x = 8, y = 4$
Number is 84

B) Solving by Elimination (Basic)

Solve the following pairs of simultaneous equations using the elimination method
$2x - 3y = 18$
$3x + 2y = 1$

C) Solving by Substitution (Basic)

Solve the following pairs of simultaneous equations using the substitution method
$-2x + 9y = 8$
$4x + 7y = 9$

D) Solving Simultaneous Equations Example (Intermediate/Advanced)

Solve the following pairs of simultaneous equations.
$\frac{5}{x} - \frac{2}{y} = 16$ **- - - Eqn 1**
$\frac{2}{x} + \frac{3}{y} = 14$ **- - - Eqn 2**

E) Simple word problem Example (Basic)

Mr McDonald has some chickens and cows in his farm. If he counted a total of 94 legs and 30 heads, find the number of chickens and the number of cows in his farm.

F) Solving Single-line Simultaneous Equations Example (Intermediate)

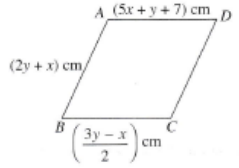
Solve the following pairs of simultaneous equations
$\frac{1}{2}(x + 3y) = 3(2y - 3x - 1) = x + y + 2$

G) Given Solutions find Unknowns (Example) (Intermediate)

If $x = 3$ and $y = -1$ are the solutions to the simultaneous equations:
$ax - by = 11$
$3x + by = 5a$
Find the values of a and b.

H) Simultaneous on Figures (Example) (Intermediate)

Find the values of x and y.



ABCD is a rhombus.

I) Money Example (Intermediate, **Important)

Sandra paid for a dress that costs $\$125$ using five-dollar and ten-dollar notes. She used 16 notes altogether. Using algebraic method, find the number of five-dollar notes she used.

J) Two-Digit Number (Example 1) (*Advanced, **Important)

A two digit number is 4 times the sum of its digits. If the digits are reversed, the number will be increased by 27. Find the number using an algebraic method.

K) Two-Digit Number (Example) (*Advanced, **Challenging)

A two digit number is such that the sum of its digits is one-seventh of the number. If the digits are reversed, the number will be decreased by 36. What is the number?