Kinematics

1 The velocity, v m/s, of a particle moving in a straight line, t seconds after leaving a fixed point O is given by $v = t^2 + kt + 12$, where k is a constant. At $t = 3$s The particle rests momentarily at point M.

a) Find the other value of t where the particle is momentarily at rest.

b) Calculate the calculate the average speed of the particle for the first 6 seconds.

c) Calculate the time at which the particle passes point M again.

a) When $t = 3, v = 0$

$0 = 3^2 + k(3) + 12$

$k = -7$

$\therefore v = t^2 - 7t + 12$

$0 = t^2 - 7t + 12$

$(t-3)(t-4) = 0$

The particle is momentarily at rest when $t = 3$ and when $t = 4$

b) $s = \int t^2 - 7t + 12\, dt$

$s = \frac{t^3}{3} - \frac{7t^2}{2} + 12t + c$

When $t = 0, s = 0, \therefore c = 0$

$\therefore s = \frac{t^3}{3} - \frac{7t^2}{2} + 12t$

When $t = 3$,

$s = \frac{3^3}{3} - \frac{7(3)^2}{2} + 12(3)$

$s = 13.5m$

When $t = 4$,

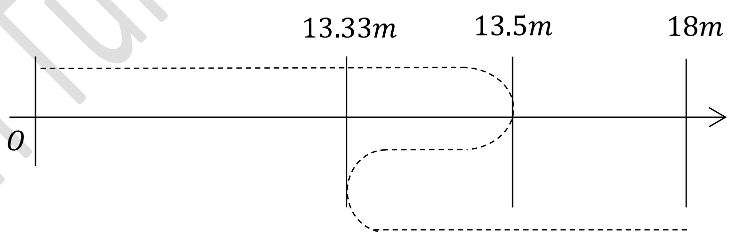
$s = \frac{4^3}{3} - \frac{7(4)^2}{2} + 12(4)$

$s = 13.33\, m$

When $t = 6$,

$s = \frac{6^3}{3} - \frac{7(6)^2}{2} + 12(6)$

$s = 18\, m$



Total distance travelled $= 13.5 + (13.5 - 13.33) + (18 - 13.33)$

$= 18.34m$

Average speed $= \frac{total\ distance}{total\ time}$

$= \frac{18.34}{6}$

$= 3.06\, m/s$

c) When $s = 13.5$,

$13.5 = \frac{t^3}{3} - \frac{7t^2}{2} + 12t$

$2t^3 - 21t^2 + 72t - 81 = 0$

By Observation, $(t-3)$ is a repeated root. Apply long division to divide by $(t-3)^2$:

$2t^3 - 21t^2 + 72t - 81 = (t-3)^2(2t-9) = 0$

$t = 3$ or $t = \frac{9}{2} = 4.5$

The particle passes M again at 4.5s

2 A particle moves in a straight line. After time t seconds, the velocity of the particle (in m/s) is $v = 16 + 4t - kt^2$, where k is a constant.

a) If the maximum velocity is 20 m/s, find the value of k.

b) Find the time when the particle is moving at its initial velocity again.

a) $v = 16 + 4t - kt^2$

$\frac{dv}{dt} = 4 - 2kt$

$0 = 4 - 2kt$

$t = \frac{4}{2k} = \frac{2}{k}$

$20 = 16 + 4\left(\frac{2}{k}\right) - k\left(\frac{2}{k}\right)^2$

$4 = \frac{8}{k} - \frac{4}{k}$

$k = \frac{4}{4} = 1$

b) When $t = 0$, $v = 16$ m/s

$16 = 16 + 4t - t^2$

$t(4 - t) = 0$

$t = 0$ (Rej) or $t = 4$

3 Two cyclists, Alvin and Bryan, are moving in the same direction on the same straight track. At a certain point O, Alvin is travelling at a speed of 20 m/s and decelerate uniformly at $4m/s^2$, overtakes Bryan who is travelling at 4 m/s and accelerating uniformly at 2 m/s^2.
a) Find the distance between Alvin and Bryan three seconds after passing O.
b) Calculate the velocity of Bryan when he overtakes Alvin.

a) Let a_A, v_A, s_A be Alvin's acceleration, velocity and displacement from O respectively
Let a_B, v_B, s_B be Bryan's acceleration, velocity and displacement from O respectively
$a_A = -4$
$v_A = \int -4\,dt = -4t + c$
When $t = 0$, $v_A = 20$,
$v_A = -4t + c$
$20 = -4(0) + c$
$c = 20$
$\therefore v_A = -4t + 20$
$s_A = \int -4t + 20\,dt$
$s_A = -2t^2 + 20t + c$
When $t = 0$, $s_A = 0$, $\therefore c = 0$
$\therefore S_A = -2t^2 + 20t$
When $t = 3$
$S_A = -2(3)^2 + 20(3) = 42$ m

$a_B = 2$
$v_B = \int 2\,dt = 2t + c$
When $t = 0$, $v_B = 4$,
$v_B = 2t + c$
$4 = 2(0) + c$
$c = 4$
$\therefore v_B = 2t + 4$
$s_B = \int 2t + 4\,dt$
$s_B = t^2 + 4t + c$
When $t = 0$, $s_B = 0$, $\therefore c = 0$
$\therefore S_B = t^2 + 4t$
When $t = 3$
$S_B = (3)^2 + 4(3) = 21$ m

Distance between Alvin and Bryan at three seconds $= 42 - 21 = 21m$
b) When $s_A = s_B$
$-2t^2 + 20t = t^2 + 4t$
$3t^2 - 16t = 0$
$t = 0$ or $t = \frac{16}{3} = 5\frac{1}{3}$
When $t = 5\frac{1}{3}$,
$v_B = 2\left(5\frac{1}{3}\right) + 4 = 14.7\ m/s$