

# Sec 1 Math: Approximation / Estimation



## A) Rounding off to required decimal place (d.p.) (Basic)

Round off 8.3796 to

- a) 1 decimal place
- b) 2 decimal place
- c) 3 decimal place
- d) nearest whole number

- a) 8.3796 = 8.4 (1 d.p.)
- b) 8.3796 = 8.38 (2 d.p.)
- c) 8.3796 = 8.380 (3 d.p.)
- d) 8.3796 = 8 (nearest whole number)

## B) Significant Figures (S.f.) Rules

**Rule 1:** All non-zero digits are significant

e.g. 51 (2 s.f.), 81.9 (3 s.f.)

**Rule 2:** Zero(s) between non-zero digits are significant

e.g. 5002 (4 s.f.), 2.034 (4 s.f.)

**Rule 3:** Zero(s) that come before the first non-zero digit are not significant

e.g. 0.0062 (2 s.f.), 0.0103 (3 s.f.)

**Rule 4:** Zero(s) following a non-zero digit after the decimal point are significant

e.g. 0.200 (3 s.f.), 82.30 (4 s.f.)

**Rule 5:** Zero(s) following a non-zero digit in a whole number may or may not be significant

e.g. 6300 (2s.f, 3s.f. or 4 s.f.), 500 (1 s.f., 2 s.f. or 3 s.f.)

## C) Identify number of S.F. (Basic)

State the number of significant figures in each of the following:

- a) 2339
- b) 0.00280
- c) 3400
- d) 210.0
- e) 20.05

- a) 2339 has 4 s.f.
- b) 0.00280 has 3 s.f.
- c) 3400 has 2 or 3 or 4 s.f.
- d) 210.0 has 4 s.f.
- e) 20.05 has 4 s.f.

## D) Rounding off to Required S.F. (Basic)

Round off each of the following to the number of significant figures as stated in the brackets.

- a) 4586 (3 s.f.)
- b) 0.05758 (2 s.f.)
- c) 3.401 (3 s.f.)
- d) 234015 (2 s.f.)
- e) 39.95 (3 s.f.)

- a) 4586 = 4590 (3 s.f.)
- b) 0.05758 = 0.058 (2 s.f.)
- c) 3.401 = 3.40 (3 s.f.)
- d) 234015 = 230000 (2 s.f.)
- e) 39.95 = 40.0 (3 s.f.)

## E) Evaluate and Round off (Basic)

Use a calculator, evaluate  $\frac{\sqrt[3]{216}-0.2033}{7.64-(1.7)^2}$  and correct the answer to 3 s.f.

$$\frac{\sqrt[3]{216}-0.2033}{7.64-(1.7)^2} \approx 0.5435059579 = 0.544 \text{ (3 s.f.)}$$

## F) Estimation

Estimation is a way to get a rough value of a calculation quickly without using a calculator.

If you see the word "estimate" in the question, make sure to:

- 1) Round off FIRST
- 2) Then do the calculation.
- 3) Round off AGAIN at the end to the desired accuracy!

*Note: "Estimate" questions always involve rounding off TWO times. Once at the beginning and once at the end.*

## G) Estimation (Without Square/Cube Root) Example (Intermediate)

Estimate the following, giving your answer corrected 1 significant figure:

- a) 34.82 – 19.35
- b) 128.1 × 0.2025
- c)  $\frac{44.73}{15.21}$

*Note: Round off to 1 digit more than what the question specify first. Since this question asks for an accuracy of 1 s.f., we will first round off the numbers to 2 s.f. before calculation.*

- a) 34.82 – 19.35 ≈ 35 – 19 (Round to 2 s.f.)  
= 16  
= 20 (1 s.f.)
- b) 128.1 × 0.2025 ≈ 130 × 0.20  
= 26  
= 30 (1 s.f.)
- c)  $\frac{44.73}{15.21} \approx \frac{45}{15}$   
= 3 (1 s.f.)

## H) Estimation (With Square/Cube Root) Example (Intermediate)

Estimate the following, giving your answer corrected 1 significant figure:

- a)  $\frac{\sqrt{35.4} \times \sqrt[3]{121}}{\sqrt{4.51}}$
- b)  $\frac{\sqrt{98-2.027^2}}{\sqrt[3]{26}}$

*Note: Values in square or cube roots, we will round the values off to the nearest perfect square and perfect cube respectively.*

- a)  $\frac{\sqrt{35.4} \times \sqrt[3]{121}}{\sqrt{4.51}} \approx \frac{\sqrt{36} \times \sqrt[3]{125}}{\sqrt{4}} = \frac{6 \times 5}{2} = 15 = 20 \text{ (1 s.f.)}$
- b)  $\frac{\sqrt{98-2.027^2}}{\sqrt[3]{26}} \approx \frac{\sqrt{100-2^2}}{\sqrt[3]{27}} = \frac{10-4}{3} = \frac{6}{3} = 2 \text{ (1 s.f.)}$

Don't write on paper but Use calculator evaluate  
 $\sqrt{35.4} = 5.95$  round off to 6,  $6^2 = 36$   
 $\sqrt[3]{121} = 4.95$  round off to 5,  $5^3 = 125$   
 $\sqrt{4.51} = 2.12$  round off to 2,  $2^2 = 4$

## I) Substitution Problems (Advanced)

Given that  $\sqrt{3.27} = 1.808$  and  $\sqrt{32.7} = 5.718$ , without using a calculator, evaluate

- a)  $\sqrt{0.327}$
- b)  $\sqrt{32700}$

*Note: Working and presentation for this question is very important. Wrong working equals zero marks even if answer is correct.*

$$\begin{aligned} \text{a) } \sqrt{0.327} &= \sqrt{\frac{32.7}{100}} \\ &= \frac{\sqrt{32.7}}{10} \\ &= \frac{5.718}{10} \\ &= 0.5718 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{32700} &= \sqrt{3.27 \times 10000} \\ &= \sqrt{3.27} \times 100 \\ &= 1.808 \times 100 \\ &= 180.8 \end{aligned}$$

## J) Reverse Approximation Problems (Advanced)

If an integer is 5000 when it is rounded off to 2 significant figures, find the difference between the maximum and minimum possible values of the integer.

The maximum possible value of the integer = 5049

The minimum possible value of the integer = 4950

The maximum difference = 5049 – 4950  
= 99

## K) Reverse Approximation Problems (Advanced)

a) The area of a rectangle is 74.5cm<sup>2</sup> when corrected to one decimal place and its breadth is 9.84cm when corrected to two decimal places. Find the greatest possible length of the rectangle, corrected to one decimal place.

b) A solid cylindrical block of wood has a radius of 28cm and a height of 60cm. Both values are measured to the nearest cm. Find the smallest possible volume of the wood correct to the nearest whole number. (Volume of cylinder =  $\pi r^2 h$ )

a) Greatest possible length of a rectangle  
 $= \frac{\text{Greatest possible area}}{\text{Smallest possible breadth}}$   
 $= \frac{74.54999}{9.835}$   
 $\approx 7.580 \text{ cm}$   
 $= 7.6 \text{ cm (1 d.p.)}$

b) Smallest possible volume of wood  
 $= \pi (\text{smallest possible } r)^2 \times (\text{Smallest possible height})$   
 $= \pi (27.5)^2 \times (59.5)$   
 $\approx 141361.8519$   
 $= 141362 \text{ cm}^3$

## Self Practice

### 1) Rounding off to required decimal place (d.p.) (Basic)

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### 3) Rounding off to Required S.F. (Basic)

Round off each of the following to the number of significant figures as stated in the brackets.

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- c) 3.401 (3 s.f.)
- d) 234015 (2 s.f.)
- e) 39.95 (3 s.f.)

### 4) Evaluate and Round off (Basic)

Use a calculator, evaluate  $\frac{\sqrt[3]{21.6} - 0.2033}{7.64 - (1.7)^2}$  and correct the answer to 3 s.f.

### 5) Estimation (Without Square/Cube Root) Example (Intermediate)

Estimate the following, giving your answer corrected 1 significant figure:

- a)  $34.82 - 19.35$
- b)  $128.1 \times 0.2025$
- c)  $\frac{44.73}{15.21}$

### 6) Estimation (With Square/Cube Root) Example (Intermediate)

Estimate the following, giving your answer corrected 1 significant figure:

- a)  $\frac{\sqrt{35.4} \times \sqrt[3]{121}}{\sqrt{4.51}}$
- b)  $\frac{\sqrt{98} - 2.027^2}{\sqrt[3]{26}}$

### 7) Substitution Problems (Advanced)

Given that  $\sqrt{3.27} = 1.808$  and  $\sqrt{32.7} = 5.718$ , without using a calculator, evaluate

- a)  $\sqrt{0.327}$
- b)  $\sqrt{32700}$

### 8) Reverse Approximation Problems (Advanced)

If an integer is 5000 when it is rounded off to 2 significant figures, find the difference between the maximum and minimum possible values of the integer.

### 9) Reverse Approximation Problems (Advanced)

a) The area of a rectangle is  $74.5\text{cm}^2$  when corrected to one decimal place and its breadth is  $9.84\text{cm}$  when corrected to two decimal places. Find the greatest possible length of the rectangle, corrected to one decimal place.

b) A solid cylindrical block of wood has a radius of 28cm and a height of 60cm. Both values are measured to the nearest cm. Find the smallest possible volume of the wood correct to the nearest whole number.  
(Volume of cylinder =  $\pi r^2 h$ )