Plane Geometry

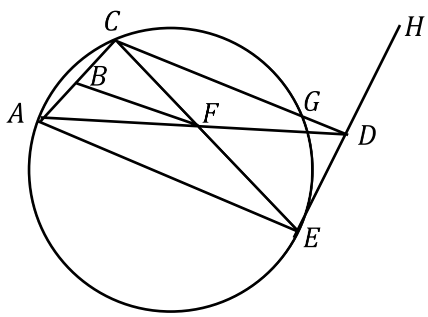
1 In the figure (not drawn to scale), AE is the diameter of the circle, F is the mid-point of CE and ED is tangent to the circle at E. CG, BF and AE are parallel lines.

a) Prove that ΔACE is congruent to ΔEDC

b) Prove that CBF is similar to DEC

c) Prove that $CD \times CE = 4CF \times BF$

d) Prove that $AE^2 = CA^2 + CD^2 + DE^2$



a) Since $CF = FE$ and $BF//AE$,

$BF = \frac{1}{2}AE$ (midpoint theorem)

$CB = BA$ (midpoint theorem)

Since $CB = BA$ and $BF//CD$,

$BF = \frac{1}{2}CD$

$\therefore CD = AE$

$\measuredangle DCE = \measuredangle AEC$ (alt $\measuredangle s$)

$CE = EC$ (Common)

$\Delta ACE \equiv \Delta EDC$ (SAS)

b) $\measuredangle CBF = \measuredangle CAE$ (Corr $\measuredangle$s)

$\measuredangle ACE = \measuredangle BCF$ (Common)

ΔCBF is similar to ΔCAE (AA)

Since $\Delta ACE \equiv \Delta EDC$ (from part a)

ΔCBF is similar to ΔDEC

c) $\frac{CF}{EC} = \frac{BF}{DC} = \frac{1}{2}$

$CD = 2BF$

$CE = 2CF$

$CD \times CE = 2BF \times 2CF = 4CF \times BF$

d) $\measuredangle AED = 90^o$ (tan⊥ radius)

$\measuredangle CDH = \measuredangle AED = 90^o$ (Corr $\measuredangle s$)

$\measuredangle ACE = 90^o$ (rt. $\measuredangle$ in semi-circle)

$AE^2 = CA^2 + CE^2$ (Pythagoras Theorem)

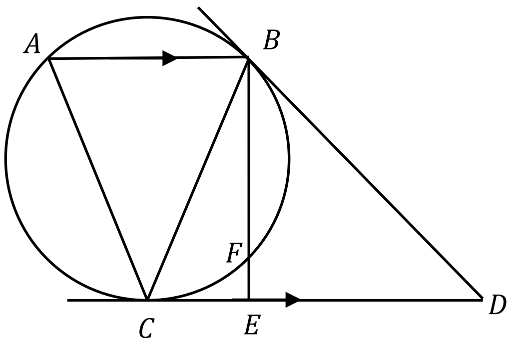
$AE^2 = CA^2 + CD^2 + DE^2$ (Pythagoras Theorem)

2 In the figure, BD and CD are tangents to the circle at B and C respectively. AB is parallel to CD and BE is perpendicular to CD.

a) Prove that ABC is an isosceles triangle

b) Prove that ΔDCB is similar to ΔCAB

c) Prove that $CE^2 + EB^2 = AB \times DB$



a) $\measuredangle ABC = \measuredangle BCE$ (alt. $\measuredangle$s)

$\measuredangle BCE = \measuredangle CAB$ (tan-chord theorem)

$\measuredangle ABC = \measuredangle CAB$

$\therefore$ ΔABC is an isosceles triangle.

b) $\measuredangle CBD = \measuredangle CAB$ (tan-chord theorem)

$\measuredangle CAB = \measuredangle CBA = \measuredangle BCD = \measuredangle CBD$ (shown in part a)

$\therefore$ ΔDCB is similar to ΔCAB (AA)

c) $\frac{CB}{AB} = \frac{DB}{CB}$ (similar Δ)

$CB^2 = AB \times DB$

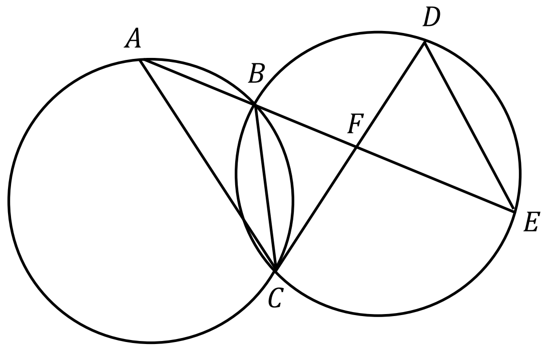
$CB^2 = CE^2 + EB^2$ (Pythagoras theorem)

$\therefore$ $CE^2 + EB^2 = AB \times DB$ (Proven)

3 In the diagram, CD is a tangent to the circle ABC at C. BC is a common chord between both circles and F is the intersection between lines AE and CD.

a) Prove that AC is parallel to DE.

b) Prove that $CF^2 = AF \times FB$



a) $\measuredangle BAC = \measuredangle BCF$ (tan chord theorem)

$\measuredangle BCF = \measuredangle BED$ ($\measuredangle s$ in same segment)

$\because \measuredangle BAC = \measuredangle BED$

AC is parallel to DE (alt $\measuredangle$s)

b) $\measuredangle FAC = \measuredangle FCB$ (tan chord theorem)

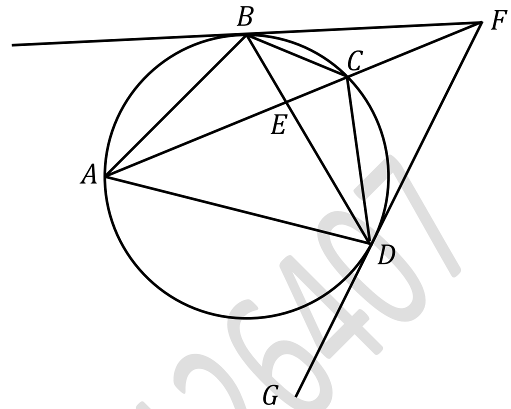
$\measuredangle BFC = \measuredangle CFA$ (common)

ΔFAC is similar to ΔFCB (AA)

$\frac{CF}{AF} = \frac{FB}{CF}$ (Corresponding sides of similar triangles)

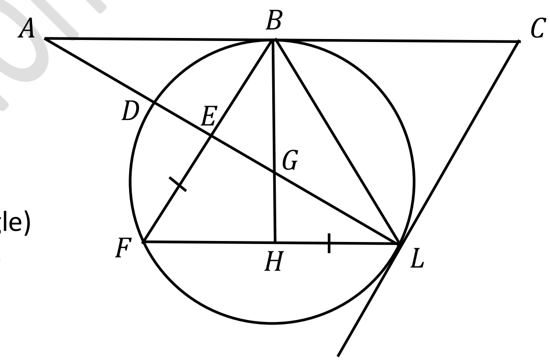
$CF^2 = AF \times FB$

4 In the diagram, BD and DF are tangents to the circle and $\measuredangle ABD = 2\measuredangle DBC$. Prove that $AD = DF$.



Let $\measuredangle DBC = x$ and $\measuredangle ABD = 2x$.
$\measuredangle ADG = \measuredangle ABD = 2x$ ($\measuredangle$ in alt segment)
$\measuredangle DAF = \measuredangle DBC = x$ ($\measuredangle$ in same segment)
$\measuredangle AFD = \measuredangle ADG - \measuredangle DAF$ (Ext $\measuredangle$ of triangle)
$= 2x - x$
$= x$
Since $\measuredangle AFD = \measuredangle DAF = x$, ΔDAF is an isosceles triangle
$\therefore AD = DF$ (sides of isosceles triangle)

5 BCF is an equilateral triangle inscribed in a circle. ABC is a tangent to the circle at point B. Given that $EF = HL$ and $AEGL$ is a straight line, prove that
a) $\Delta FEL \equiv \Delta LHB$
b) $\measuredangle BGE = \measuredangle ABE$



a) $EF = HL$ (Given)
$FL = LB$ (Sides of Equilateral triangle)
$\measuredangle EFL = \measuredangle HLB = 60^o$ ($\measuredangle s$ in equilateral triangle)
$\therefore \Delta FEL \equiv \Delta LHB$ (SAS)

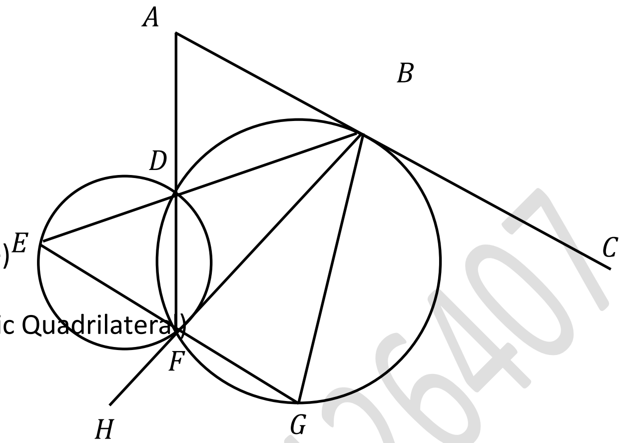
b) $\measuredangle BLG = 60^o - \measuredangle FLE$
$\measuredangle FLE = \measuredangle LBH$ (congruent triangles)
$\measuredangle BGE = \measuredangle LBH + \measuredangle BLG$ (Ext $\measuredangle$ of triangle)
$= \measuredangle FLE + (60^o - \measuredangle FLE)$
$= 60^o$
$\measuredangle ABE = \measuredangle BLF = 60^O$ ($\measuredangle$s in alt segment)
$\therefore \measuredangle BGE = \measuredangle ABE = 60^o$ (Proven)

6 In the diagram, the 2 circles intersect at points D and F. ABC is a tangent to the bigger circle at B and HFB is a tangent to the smaller circle at F. ADF, EDB and EFG are straight lines.

a) Prove that $BF = BG$.

b) Show that ABC is parallel to EFG

c) Show that ΔBDA is similar to ΔEGB



a) Let $\measuredangle BFG = x$

$\measuredangle EFH = \measuredangle BFG = x$ (Vert. Opp. ∡s)

$\measuredangle EDF = \measuredangle EFH = x$ (∡s in alt segment)

$\measuredangle BDF = 180^o - \measuredangle EDF$ (∡s on a str line)

$= 180^o - x$

$\measuredangle BGF = 180^o - \measuredangle BDF$ (Opp ∡s in Cyclic Quadrilateral)

$= 180^o - (180^o - x)$

$= x$

Since $\measuredangle BFG = \measuredangle BGF = x$,

ΔBFG is an isosceles triangle

∴ $BF = BG$ (Proven)

b) $\measuredangle ABF = \measuredangle BGF = x$ (∡s in alt segment)

Since $\measuredangle BFG = \measuredangle BGF$, (shown in part a)

$\measuredangle ABF = \measuredangle BFG = x$

∴ ABC is parallel to EFG (alternate ∡s)

c) $\measuredangle ABD = \measuredangle BEG$ (alt ∡s)

$\measuredangle BDF = 180^o - \measuredangle BGF$ (Opp ∡s in Cyclic Quadrilateral)

$\measuredangle BDA = 180^o - \measuredangle BDF$ (∡s on a str line)

$= 180^o - (180^o - \measuredangle BGF)$

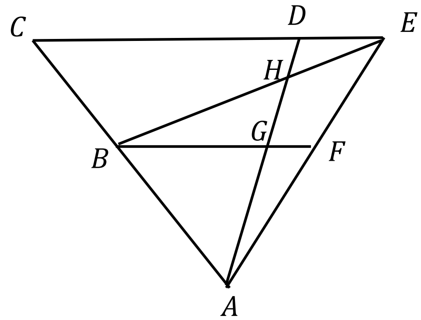
$= \measuredangle BGF$

∴ ΔBDA is similar to ΔEGB (AA)

7 In the diagram, B is the mid-point of AC and G is the midpoint of AD. CDE and BGF are straight lines. Given that $BH = 2HE$, show that,

a) ΔDHE is similar to ΔGHB

b) $AH = 5DH$



a) $CD//BG$ (mid point theorem)

$\measuredangle EDH = \measuredangle BGH$ (alt $\measuredangle$s)

$\measuredangle DHE = \measuredangle GHB$ (vertically Opp $\measuredangle s$)

ΔDHE is similar to ΔGHB (AA)

b) $\frac{BH}{HE} = \frac{GH}{HD} = \frac{2}{1}$

$DH = \frac{1}{3}DG$

$DH = \frac{1}{3}\left(\frac{1}{2}DA\right)$

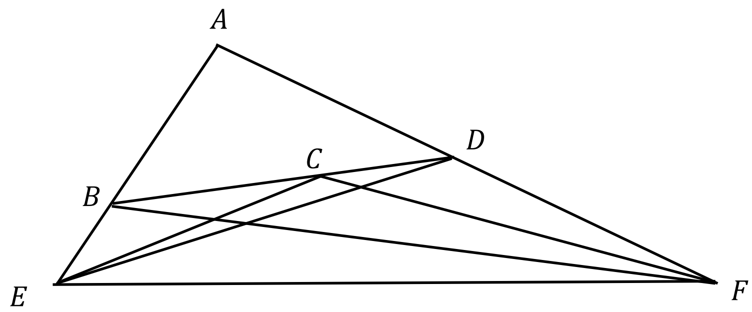
$DH = \frac{1}{6}DA$

$DH = \frac{1}{6}(DH + AH)$

$6DH = DH + AH$

$AH = 5DH$

8 The diagram shows triangle AEF where $AD = DF$, $AB = 2BE$ and $BC = 3CD$. Find the value of $\frac{Area\ of\ \Delta CDE}{Area\ of\ \Delta AEF}$.



$Area\ of\ CDE = \frac{1}{4} \times Area\ of\ \Delta BDE$

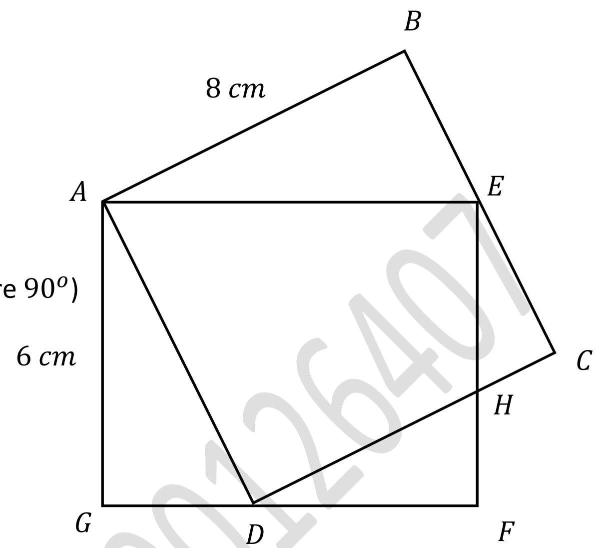
$= \frac{1}{4}\left(\frac{1}{3} \times Area\ of\ \Delta ADE\right)$

$= \frac{1}{12}\left(\frac{1}{2} \times Area\ of\ \Delta AEF\right)$

$= \frac{1}{24} \times Area\ of\ \Delta AEF$

$\frac{Area\ of\ \Delta CDE}{Area\ of\ \Delta AEF} = \frac{1}{24}$

9 In the figure, $ABCD$ is a square with sides $8\ cm$ and $AEFG$ is a rectangle. Given that $AG = 6cm$, find the length of GF.



Let $\measuredangle GAD = x$

$\measuredangle DAE = 90^o - \measuredangle GAD$

$= 90^o - x$

$\measuredangle BAE = 90^o - \measuredangle DAE$

$= 90^O - (90^o - x)$

$= x$

$\therefore \measuredangle GAD = \measuredangle BAE = x$

$\measuredangle ABE = \measuredangle AGD = 90^O$ ($\measuredangle s$ of square and rectangle are 90^o)

$\therefore \Delta GAD$ is similar to ΔBAE (AA)

$AD = AB = 8cm$ (Sides of square are equal)

$\frac{GA}{BA} = \frac{AD}{AE}$

$\frac{6}{8} = \frac{8}{AE}$

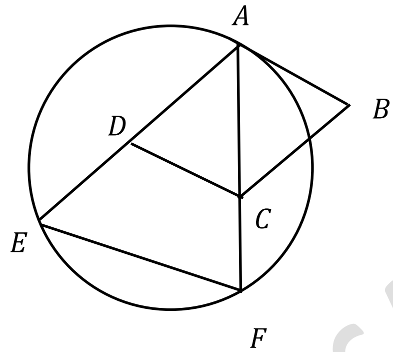
$AE = \frac{64}{6} = 10.7\ cm$

$GF = AE = 10.7\ cm$ (Opposite sides of rectangle are equal)

10 In the diagram, AFE is a triangle inscribed in a circle. $ABCD$ is a parallelogram and AB is a tangent to the circle at point A. Prove that

a) ΔFAE is similar to ΔBCA

b) $AC \times AF = AD \times AE$



a) $\measuredangle BCA = \measuredangle DAC$ $(alt\ \measuredangle s)$

$\measuredangle CAB = \measuredangle AEF$ $(\measuredangle s\ in\ alt\ segment)$

$\therefore$ ΔFAE is similar to ΔBCA (AA)

b) $\measuredangle DCA = \measuredangle CAB$ $(alt\ \measuredangle s)$

$\measuredangle AEF = \measuredangle CAB$ $(\measuredangle s\ in\ alt\ segment)$

$\therefore \measuredangle DCA = \measuredangle AEF$

$\measuredangle DAC = \measuredangle FAE$ (Common $\measuredangle$)

$\therefore$ ΔDAC is similar to ΔFAE (AA)

$\frac{AC}{AE} = \frac{AD}{AF}$

$AC \times AF = AD \times AE$ (Proven)