(3) Polynomials

1. Without using long division, find the remainder when $2\,x^6 + x^4 - 15\,x^2 - 14$ is divided by $x^2 + 2$.

2. A cubic polynomial, $f(x)$, leaves a remainder of 12 when divided by x and $f(x+1) - f(x-1) \equiv 12x^2 - 12x - 42$.By substituting suitable values of x,
   a) Find the remainder when $f(x)$ is divided by $(x-2)$
   b) Show that $f(-2) = 30$
   c) Show that $(x-4)$ is a factor of $f(x)$.

3. Given that $(x-1)(x-2)(Ax+B) + C(x-2) + D = 3\,x^3 - 7\,x^2 + 3\,x + 2$ for all values of x, find A, B, C and D.

4. $(x-2)$ is a factor of $g(x) + 5$, where $g(x)$ is a polynomial. Find the remainder when $f(x) = (2x^3 + 3x^2 - 4)g(x)$ is divided by $(x-2)$.

5. The term containing the highest power of x in the polynomial $f(x)$ is x^4 and the roots of $f(x) = 0$ are -6 and 3. $f(x)$ has a remainder of -84 when divided by $(x-1)$ and a remainder of -96 when divided by $(x-2)$. Find the expression for $f(x)$.

6. Express $\frac{3x^2+5}{x^4-1}$ in partial fractions

7. Given that $f(x) = 4x^3 - 2x^2 + 5x - 1$, find
   i) the remainder when $f(x)$ is divided by $(x-1)$
   ii) the remainder when $f(x-8)$ is divided by $(x-9)$.
   iii) deduce the remainder when $f(x^2-6)$ is divided by (x^2-8).

8. When the function $f(x)$ is divided by $(x+1)$, the remainder is -5. When $f(x)$ is divided by $(x-1)$, the remainder is -1. When $f(x)$ is divided by (x^2-1), the remainder is $(Ax+B)$. Find A and B.

9. $f(x)$ is a function where $f(x) = ax^3 + bx^2 + 2x - 5$. $2f(x) - 6$ is divisible by $(x-1)$ and when $f(x) + 4$ is divided by $(x+2)$, it leaves a remainder of -5. Find A and B.

10. Given that (x^2-3) is a factor of $f(x) = x^3 + ax^2 + bx - 3$
    i) Find the value of a and b.
    ii) Hence, factorize $f(x)$ completely.
    iii) Hence, solve the equation $1 + ay + by^2 - 3y^3 = 0$.