

### (13) Integration

1. Integrate the following:

a)  $\int \frac{2x}{2-3x^2} dx$

b)  $\int \frac{3-x}{1-x} dx$

2. a) Show that  $\frac{d}{dx} \ln\left(\frac{x^2}{\sin x}\right) = \frac{2}{x} - \cot x$ .

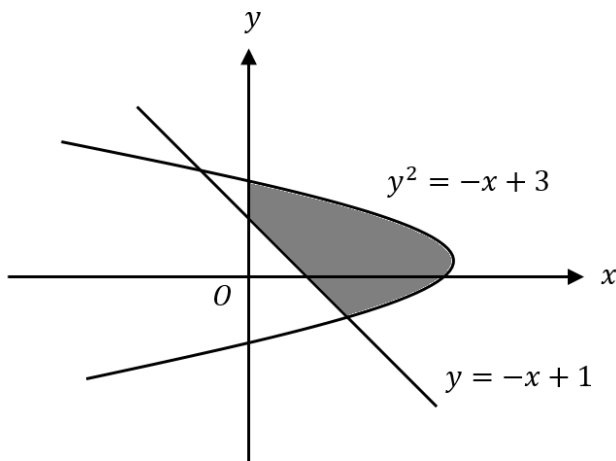
b) Hence, find  $\int \frac{1}{2} \cot x dx$ .

3. a) Express  $\frac{x^3-x^2+3x+9}{x(x^2+3)}$  in partial fractions.

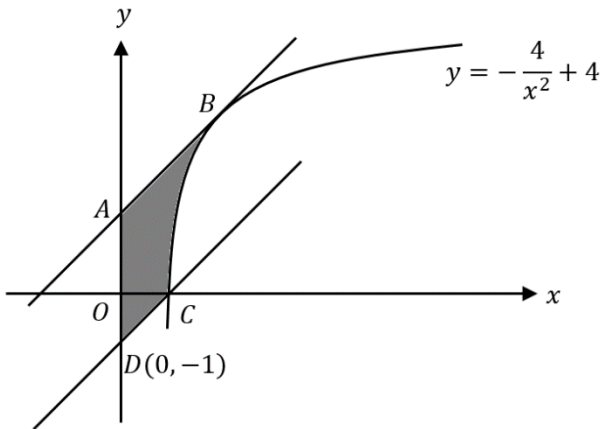
b) Differentiate  $\ln(x^2 + 3)$  with respect to  $x$ .

c) Use the results from parts (a) and (b) to find  $\int_1^2 \frac{x^3-x^2+3x+9}{x(x^2+3)} dx$ .

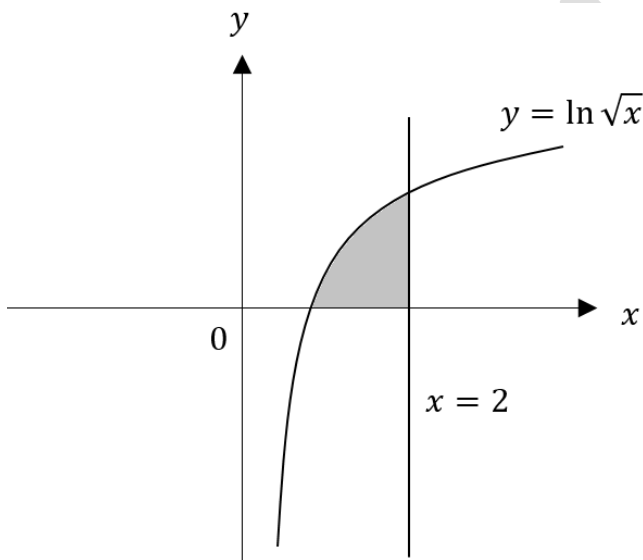
4. The diagram shows the curve  $y^2 = -x + 3$  and the line  $y = -x + 1$ . Find the area of the shaded region.



5. The diagram shows the curve  $y = -\frac{4}{x^2} + 4$ . AB is the tangent to the curve and is parallel to CD. D is the point (0,-1).  
 i) Find the gradient of CD  
 ii) Find the coordinates B and A  
 iii) Find the area of the shaded region ABCD.



6. a) The shaded region below is bounded by the curve  $y = \ln \sqrt{x}$ , the x-axis and the line  $x = 2$ . Show that the area of the shaded region below is,  $A = \ln 2 - \frac{1}{2}$ .



7. The gradient function of a curve is given by  $3x^2 + 3x - 18$ . The curve has a maximum value of  $K$  and a minimum value of  $-12$ . Find the value of  $K$ .
8. A curve has a turning point at  $(2, -10)$  and  $\frac{d^2y}{dx^2} = 12x - 6$ . Find the equation of the curve.
9. The gradient of a function of a curve is given by  $\frac{40}{(x-3)^3} - 2$ . Given that the line  $7y = x + 20$  is a normal to the curve, find the equation of the curve.
10. Solve the following indefinite integrals:
- i)  $\int \sin^2 2x + \frac{1}{\cos^2(x+\frac{\pi}{6})} dx$
- ii)  $\int \tan^2\left(x + \frac{\pi}{3}\right) + \cos x \sin x dx$