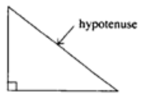
Sec 2 Math: Pythagoras Theorem

A) Hypotenus

In a right angled triangle, the side opposite the right angle, which is also the longest side of the triangle is called the hypotenuse.



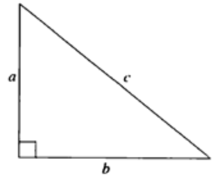
B) Pythagoras' Theorem

Pythagoras Theorem:

For a right-angled triangle,

$c^2 = a^2 + b^2$

where c is the length of the hypotenuse.



C) Common Misconception

$a^2 = b^2 + c^2$ Correct

$a = \sqrt{b^2 + c^2}$

$a = b + c$ Wrong

***Please note that:*

$\sqrt{b^2 + c^2}$ **is NOT equal to** $b + c$

$\sqrt{(b + c)^2}$ **is equal to** $b + c$

D) Common Pythagorean Triples (Optional, Extra knowledge)

Pythagorean triples are integer side lengths of a right angle triangle that satisfies the Pythagoras' Theorem

Common Pythagorean triples are

a) $[3, 4, 5]$ and its multiples (i.e. [6, 8, 10] or [30,40,50])

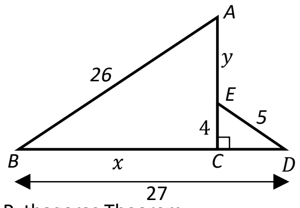
b) [5, 12, 13]

c) [8, 15, 17]

E) Pythagoras Example (Intermediate)

i) Find x and y.

ii) Find shortest distance from C to AB



i) By Pythagoras Theorem,

$CD^2 + 4^2 = 5^2$

$CD = \sqrt{5^2 - 4^2} = 3$

$x = 27 - 3 = 24$

By Pythagoras Theorem,

$AC^2 + 24^2 = 26^2$

$AC = \sqrt{26^2 - 24^2} = 10$

$y = 10 - 4 = 6$

ii) Area of triangle ABC $= \frac{1}{2} \times 24 \times 10$

$= 120\ units^2$

Area of triangle ABC $= \frac{1}{2} \times 26 \times h$

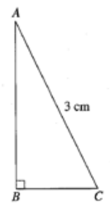
$120 = \frac{1}{2} \times 26 \times h$

$h = \frac{12}{13}$

**Note: for Part (ii), we used the area formula twice for triangle ABC but each time we took different lengths as the base and height.*

F) Pythagoras Example (Intermediate)

In ΔABC, $AB = 2BC$ and $AC = 3cm$. Find BC.



Let BC be x.

$\therefore AB = 2x$

By Pythagoras' Theorem,

$x^2 + (2x)^2 = 3^2$

$5x^2 = 9$

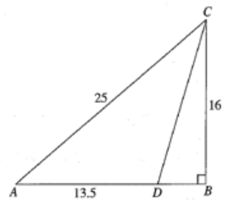
$x^2 = \frac{9}{5}$

$x = \sqrt{\frac{9}{5}}$ or $x = -\sqrt{\frac{9}{5}}$ (Rej since $x > 0$)

$x = 1.34\ cm$ (3 s.f.)

G) Common Mistake!

Find length of CD.



$CD^2 + 13.5^2 = 25^2$ (*Common ERROR!!)

*** The above statement is wrong because ACD is not a right-angled triangle. Pythagoras can only be applied to right-angled triangle!*

H) Proving Right Angles using Converse of P. Theorem (Intermediate)

(**Presentation Important !!!)

Determine whether triangle ABC is a right-angled triangle.

a) ΔABC, AB= 41 cm, $BC = 40$ cm and $AC = 9$ cm.

b) ΔXYZ, XY= 15 cm, $YZ = 16$ cm and $XZ = 20$ cm.

a) $BC^2 + AC^2 = 40^2 + 9^2$

$= 1681$

$AB^2 = 41^2$

$= 1681$

Since $BC^2 + AC^2 = AB^2$, by the converse of Pythagoras' theorem, ΔABC is a right-angled triangle.

b) $XY^2 + YZ^2 = 15^2 + 16^2$

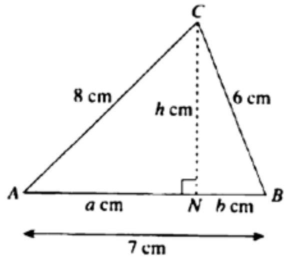
$= 481$

$XZ^2 = 20^2$

$= 400$

Since $XY^2 + YZ^2 \neq XZ^2$, by the converse of Pythagoras' theorem, ΔXYZ is **NOT** a right-angled triangle.

I) Pythagoras with Algebraic Identities (Intermediate)



i) express a^2 in terms of h,

ii) express b^2 in terms of h,

iii) find the value of $a^2 - b^2$,

iv) find the value of $a - b$.

i) $a^2 = 8^2 - h^2$

ii) $b^2 = 6^2 - h^2$

iii) $a^2 - b^2 = (8^2 - h^2) - (6^2 - h^2)$

$= 8^2 - 6^2$

$= 28$

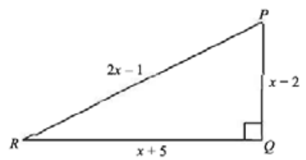
iv) $a^2 - b^2 = (a + b)(a - b)$

$28 = (7)(a - b)$

$a - b = 4$

J) Pythagoras with Quadratic Eqn (Intermediate)

Find the value of x.



By Pythagoras' Theorem,

$(2x - 1)^2 = (x + 5)^2 + (x - 2)^2$

$4x^2 - 4x + 1 = x^2 + 10x + 25 + x^2 - 4x + 4$

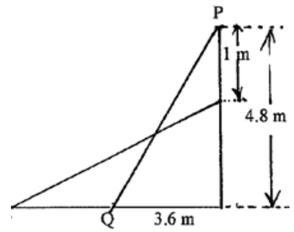
$2x^2 - 10x - 28 = 0$

$2(x - 7)(x + 2) = 0$

$x = 7$ or $x = -2$

K) Pythagoras Real-life Application (Intermdiate)

In the diagram below, a ladder PQ, leans against a vertical wall. The foot of the ladder is initially 3.6m from the wall and the ladder reaches a height of 4.8 m. The ladder then slides 1m down the wall. Find the distance moved by the foot of the ladder.



**Key is to note that the length of the ladder does not change.*

$PQ = \sqrt{4.8^2 + 3.6^2}$ (By Pythagoras Theorem)

$PQ = 6$ m

The ladder is 6m long

New height of ladder $= 4.8 - 1 = 3.8$ m

Distance of foot from wall $= \sqrt{6^2 - 3.8^2}$

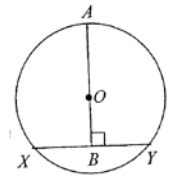
$= 4.64$m (3 s.f.)

Distance moved by foot $= 4.64 - 3.6$

$= 1.04$m (3 s.f.)

L) Pythagoras (Advanced)

In the figure, O is the centre of the circle. AB is a line passing through O and perpendicular to XY. If $XY = 3cm$ and $AB = 3.5cm$, calculate the radius of the circle.



Let the radius of circle be r.

$OA = OX = OY = r$

$OB = 3.5 - r$

$OX = r$

$XB = \frac{3}{2} = 1.5$

By Pythagoras Theorem,

$XB^2 + OB^2 = OX^2$

$1.5^2 + (3.5 - r)^2 = r^2$

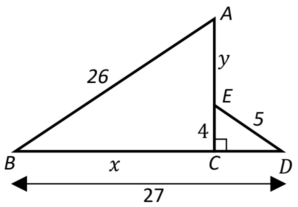
$2.25 + 12.25 + r^2 - 7r = r^2$

$7r = 14.5$

$r = 2.07$ (3 s.f.)

Radius of the circle is 2.07 cm

i) Find x and y.
ii) Find shortest distance from C to AB



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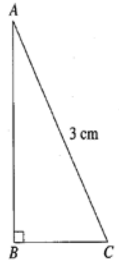

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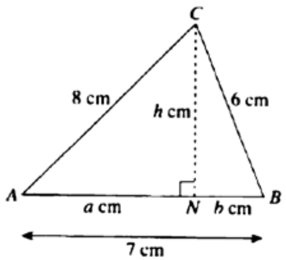
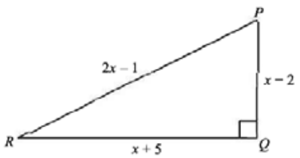


Find the value of x.



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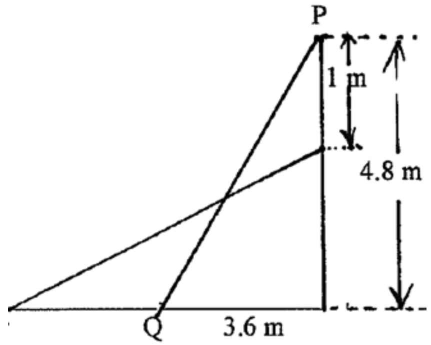

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