Sec 1 Math: Number Pattern

A) Number Pattern/Sequence

In a sequence of number, each number in the number sequence is called a **term**.

The first number is the 1st term, second number is 2nd term and the n^{th} number is called the $\boldsymbol{n^{th}}$ **term**.

$\boldsymbol{n}$ represents the **position of the term** in the General term formula.

B) General Formula for Common Difference

$T_n = Dn + Bt$ **(*"Duck Noodles + Bubble tea"*)**
("Deez Nutz + Bubble tea")

D is the common difference
$\boldsymbol{Bt}$ **is the "Before-Term" (i.e. the term before the first term)**

Example: Pattern 3, 7, 11, 15, 19, …
$D = 4$, $Bt = -1$
$T_n = Dn + Bt$
$\therefore n^{th}\ term\ formula = 4n - 1$

C) Common Difference Example (Basic)

Consider the sequence $2, 8, 14, 20$, …
a) Write down an expression, in terms of n, for the n^{th} term of the sequence.
b) Find the 31^{st} term of the sequence.
c) Is 298 in the sequence? Explain.

a) $T_n = Dn + Bt$
$T_n = 6n - 4$

b) $T_n = 6n - 4$, Sub $n = 31$,
$T_{31} = 6(31) - 4$
$T_{31} = 182$

c) $T_n = 6n - 4$, Sub $T_n = 298$,
$298 = 6n - 4$
$302 = 6n$
$n = 50.33$
Since n is not an integer, 298 is not in the sequence.

D) Questions with Given nth term Formula

The n^{th} term of a sequence is given by $\frac{n}{2n+1}$. Write down the first three terms of the sequence.

First term $(sub\ n = 1) = \frac{(1)}{2(1)+1} = \frac{1}{3}$
Second term $(sub\ n = 2) = \frac{(2)}{2(2)+1} = \frac{2}{5}$
Third term $(sub\ n = 3) = \frac{(3)}{2(3)+1} = \frac{3}{7}$
First three terms are: $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$.

E) Common Patterns

Write down the next two terms of the following sequences:
a) $8, 5, 2, -1, \ldots, \ldots$
b) $1, 8, 27, 64, \ldots, \ldots$
c) $\frac{3}{7}, \frac{6}{13}, \frac{9}{19}, \frac{12}{25}, \ldots, \ldots$
d) $1, 1, 2, 3, 5, 8, \ldots, \ldots$
e) $4, 9, 16, 25, \ldots, \ldots$
f) $\frac{1}{3}, \frac{4}{7}, \frac{11}{18}, \frac{29}{47}, \ldots, \ldots$
g) $\frac{1}{3}, 1, 3, 9, 27, \ldots, \ldots$

a) $-4, -7$ (Common difference -3)
b) $125, 216$ (Perfect Cubes)
c) $\frac{15}{31}, \frac{18}{37}$ (Top is +3, btm is +6)
d) $13, 21$ (Sum of two previous terms)
e) $36, 49$ (Perfect Squares)
f) $\frac{76}{123}, \frac{199}{322}$ (Top: Previous top + Previous btm)
(Btm: Previous btm + current Top)
g) 81, 243 (Previous term multiply by 3)

F) Number Pattern Example (Intermediate)

a) Consider the sequence $1, 4, 9, 16$, …
i) Write down the next two terms in the sequence.
ii) Write down an expression for the n^{th} term.
iii) Explain whether 16900 is in the sequence.
b) Consider another sequence $2, 6, 12, 20$, …
Use the sequence in (a) or otherwise, write down an expression for the n^{th} term.

ai) $25, 36$
aii) n^2
aiii) $\sqrt{16900} = 130$.
Since 16900 is a perfect square, it is in the sequence.
It is the 130^{th} term.
b) $n^2 + n$ (by observation)

G) Line Pattern Example (Intermediate)

Consider the pattern:

Line 1:	$3^2 + 4^2$	$= 5^2$
Line 2:	$5^2 + 12^2$	$= 13^2$
Line 3:	$7^2 + 24^2$	$= 25^2$
Line 4:	$9^2 + 40^2$	$= 41^2$
Line 5:	$11^2 + 60^2$	$= 61^2$
…	…	…
Line n:	$p^2 + q^2$	$= r^2$

a) Write down the 6^{th} line of the number pattern.
b) Write a formula for p in terms of n.
c) Write a formula for q in terms of n.
d) Hence, write a formula for r^2 in terms of n.
e) Write down the 10^{th} line of the number pattern.

a) Line 6: $13^2 + 84^2 = 85^2$

b) $p = Dn + Bt$
$p = 2n + 1$

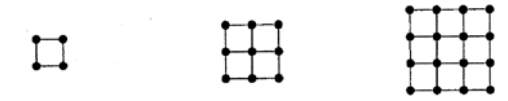
c) $q = Xn^2 + Yn + Z$
*(*Refer to next page last box (J and K) for method to get general formula for increasing difference pattern)*
$q = 2n^2 + 2n$

d) $r^2 = p^2 + q^2$
$r^2 = (2n+1)^2 + (2n^2 + 2n)^2$
(*For sec 2 and above students, please expand and simplify this formula)

e) Sub $n = 10$:
Line 10: $21^2 + 220^2 = 221^2$

H) Diagram Pattern Example (Intermediate)

John makes fenses using identical metal rods that are one metre long. The rods are bolted together at their ends. Some fences, with different lengths, are shown below.



Length = 1 m Length = 2 m Length = 3 m

• Shows the position of a bolt

The table below shows the number of bolts and rods used for various lengths of fence.

Length (m)	1	2	3	4	…	n
No. of bolts	4	9	16	x	…	B
No. of rods	4	12	24	y	…	R

a) Write down the values of x and y
b) Write down an expression for:
i) B in terms of n
ii) R in terms of n
c) John has 600 bolts. How many complete 6 m long fences can he make.

a) $x = 25,\ y = 40$
bi) $B = (n+1)^2$
bii) $R = 2n(n+1)$
c) Sub $n = 6$,
$B = (6+1)^2 = 49$
$\frac{600}{49} = 12.24$
John can make 12 complete 6m long fences

I) Line Pattern Example (advanced)

Consider the pattern on the right:
a) Write down the next line.
b) Find an expression in terms of n to find value of T_n
c) Find an expression for $T_{n+1} - T_n$ ***(*part c for upper sec only)***

$\boldsymbol{T_1 = 3^0 + 1 + 2^2 = 6}$
$\boldsymbol{T_2 = 3^1 + 4 + 3^2 = 16}$
$\boldsymbol{T_3 = 3^2 + 7 + 4^2 = 32}$
$\boldsymbol{T_4 = 3^3 + 10 + 5^2 = 62}$

a) $T_5 = 3^4 + 13 + 6^2 = 130$
b) $T_n = 3^{n-1} + 3n - 2 + (n+1)^2$
c) $T_{n+1} = 3^{(n+1)-1} + 3(n+1) - 2 + ((n+1)+1)^2$
$T_{n+1} - T_n = \left[3^{(n+1)-1} + 3(n+1) - 2 + \left((n+1)+1\right)^2\right] - [3^{n-1} + 3n - 2 + (n+1)^2]$
$= 2(3^{n-1} + n + 3)$ (After expanding and simplifying)

J) Comparing between Patterns (Intermediate)

a) The n^{th} term of a sequence is given by $3n^2 + 7$. Write down the first four terms of the sequence.

b) The first four terms of another sequence are $15, 24, 39, 60$. By comparing this sequence with the sequence in (a), find the expression, in terms of n, for the n^{th} term of this sequence.

a) $1^{st}\ term : 3(1)^2 + 7 = 10$
$2^{nd}\ term : 3(2)^2 + 7 = 19$
$3^{rd}\ term : 3(3)^2 + 7 = 34$
$4^{th}\ term : 3(4)^2 + 7 = 55$

The first four terms of the sequence are $10, 19, 34, 55$.

b)
***Note**: This question requires us to compare between two sequence and not just look within one sequence alone.*
Sequence found in part (a): $10, 19, 34, 55$
Sequence given in part (b) : $15, 24, 39, 60.$ ← +5

Observe that values in part (b) are obtained by adding 5 to the corresponding values in part (a).
Since the formula for part (a) is given as $3n^2 + 7$
The formula for part (b) is $3n^2 + 7 + 5$
Expression for sequence is $3n^2 + 12$

J) Common Difference of Difference General formula (*Advanced, more applicable for Upper sec)

Consider the following number sequence : $3, 9, 17, 27, 39, \ldots$
Write down an expression, in terms of n, for the n^{th} term of the sequence
**Note that the difference is increasing by 2 for each subsequent term.*

METHOD 1: Calculator method for *Casio Fx 96-SG Plus*

1) Press "Mode", "3", "2"
2) Fill up the table of numbers with the following

Fixed values

$$\begin{pmatrix} 1 & 1 & 1 & \mathbf{3} \\ 4 & 2 & 1 & \mathbf{9} \\ 9 & 3 & 1 & \mathbf{17} \end{pmatrix}$$

3) Press Equals and Record values of $X, Y\ and\ Z$.
In this case, calculator shows: $X = 1, Y = 3, Z = -1$
4) The General term formula is in the form $T_n = Xn^2 + Yn + Z$.
Substitute the calculator values into the $X, Y\ and\ Z$ in the General term formula.
$\therefore T_n = 1n^2 + 3n - 1$
5) Do a check to verify that formula works.
Sub $n = 4$ into formula:
$T_4 = 1(4)^2 + 3(4) - 1$
$T_4 = 27$ (Formula is correct since the forth term in the pattern is 27)

METHOD 2: Calculator method for *Casio Fx 97SG X*

1) Press "Menu", "3", "3"
2) Fill up the table of numbers with the following

X	Y
1	**3**
2	**9**
3	**17**

3) Press "OPTN", "4".
Copy down the equation shown and sub in the values to "a","b","c" as shown in calculator. Replace the alphabet "x" with "n"
$T_n = -1 + 3n + 1n^2$
4) Do a check to verify that formula works.
Sub $n = 4$ into formula:
$T_4 = -1 + 3(4) + 1(4)^2$
$T_4 = 27$ (Formula is correct since the forth term in the pattern is 27)

METHOD 3: Formula Method (Without Calculator)

$$T_n = a + d(n-1) + \frac{c}{2}(n-1)(n-2)$$

***Where **a** is the first term, **d** is the first difference and **c** is the difference of the difference.*
In the above pattern, $a = 3, d = 6, c = 2$.
$T_n = 3 + 6(n-1) + \frac{2}{2}(n-1)(n-2)$
$T_n = 3 + 6n - 6 + n^2 - 3n + 2$
$T_n = n^2 + 3n - 1$

K) Another Example to further Check understanding (*Advanced, Upper sec)

Write down an expression, in terms of n, for the n^{th} term of the sequence $5, 8, 14, 23, 35, \ldots \ldots$

Calculator method for *Casio Fx 96-SG Plus*
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$$\begin{pmatrix} 1 & 1 & 1 & \mathbf{5} \\ 4 & 2 & 1 & \mathbf{8} \\ 9 & 3 & 1 & \mathbf{14} \end{pmatrix}$$

3) Press Equals and Record values of $X, Y\ and\ Z$.
In this case, calculator shows: $X = \frac{3}{2}, Y = -\frac{3}{2}, Z = 5$
$\therefore T_n = \frac{3}{2}n^2 - \frac{3}{2}n + 5$

Calculator method for *Casio Fx 97SG X*
1) Press "Menu", "3", "3"
2) Fill up the table of numbers with the following

X	Y
1	**5**
2	**8**
3	**14**

3) Press "Menu", "4".
Copy down equation given
$\therefore T_n = 1.5n^2 - 1.5n + 5$

Common Difference Example (Basic)

Consider the sequence $2, 8, 14, 20, \ldots$

a) Write down an expression, in terms of n, for the n^{th} term of the sequence.

b) Find the 31^{st} term of the sequence.

c) Is 298 in the sequence? Explain.

Questions with Given nth term Formula (Basic)

The n^{th} term of a sequence is given by $\frac{n}{2n+1}$. Write down the first three terms of the sequence.

Common Patterns

Write down the next two terms of the following sequences:

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c) Write a formula for q in terms of n. *(*For Sec 2 and above)*

d) Hence, write a formula for r^2 in terms of n. *(*For Sec 2 and above)*

e) Write down the 10^{th} line of the number pattern. *(*For Sec 2 and above)*

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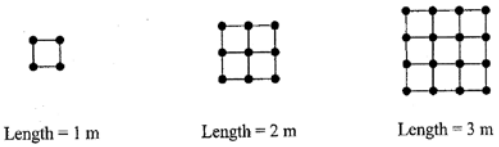

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c) Find an expression for $T_{n+1} - T_n$ *(*for upper sec only)*

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