Sec 2 Math: Further Expansion/Factorization

A) Expansion

i) $a(b+c) = ab + ac$

ii) $(a+b)(c+d) = ac + ad + bc + bd$

iii) $(a+b)(c+d+e) = ac + ad + ae + bc + bd + be$

B) Expansion Example (Basic)

Simplify the following expressions:

$\mathbf{2(2x+y)(x-3y) - (3x-y)(2x-3y)}$

Note: "Simplify" means expand and simplify

Put big bracket after expansion

$2(2x+y)(x-3y) - (3x-y)(2x-3y)$
$= 2(2x^2 - 6xy + xy - 3y^2) - (6x^2 - 9xy - 2xy + 3y^2)$
$= 4x^2 - 12xy + 2xy - 6y^2 - 6x^2 + 9xy + 2xy - 3y^2$
$= 4x^2 - 6x^2 - 6y^2 - 3y^2 - 12xy + 2xy + 9xy + 2xy$
$= -2x^2 - 9y^2 + xy$

Negative before bracket causes all signs to change

C) Special Identities

(Very Important, Memorize!)

$(a+b)^2 = a^2 + b^2 + 2ab$
$(a-b)^2 = a^2 + b^2 - 2ab$
$(a+b)(a-b) = a^2 - b^2$

D) Factorization by Grouping

$ax + ay + bx + by = a(x+y) + b(x+y)$
$= (x+y)(a+b)$

Usually, when there are four terms and we need to factorize them, we factorize by grouping.

E) Factorization of Quadratic Expressions

Factorize $\mathbf{6a^2 + 5a - 6}$

$6a^2 + 5a - 6$ is a quadratic expression and has three terms, we can use either the crossbox or multiplication frame to factorize it.

Method 1 (Cross Box)

$3a$ ×	-2	$-4a$
$2a$ ×	3	$9a$
$6a^2$	-6	$5a$

$\therefore 6a^2 + 5a - 6 = (3a-2)(2a+3)$

Method 2 (Multiplication Frame)

	$2a$	3
$3a$	$6a^2$	$9a$
-2	$-4a$	-6

$\therefore 6a^2 + 5a - 6 = (3a-2)(2a+3)$

F) Factorization Examples (Intermediate)

Factorise completely

a) $4x^2y - 25z^2y$
b) $2p^2 - 3pq - 5q^2$
c) $6xp - 9xq + 12yq - 8yp$
d) $2a^3 - 32a$
e) $x^4 - 81y^4$
f) $32y^4 - 2$
***g) $5x^2 + 5y^2 - 10xy - 45$** **(advanced)**
***h) $5m(m^2-4) - m^2 - 2m$** **(advanced)**

*For all factorization questions:
Step 1: Check for Common terms and factorize
Step 2: 2 terms (Usually factorize using $a^2 - b^2$)
3 terms (Usually factorize using crossbox)
4 terms (Usually factorize by grouping)
Step 3: Check whether can factorize further.

a) $4x^2y - 25z^2y = y(4x^2 - 25z^2)$
$= y((2x)^2 - (5z)^2)$
$= y(2x+5z)(2x-5z)$

b) $2p^2 - 3pq - 5q^2$

p	q	$2pq$
$2p$	$-5q$	$-5pq$
$2p^2$	$-5q^2$	$-3pq$

$2p^2 - 3pq - 5q^2 = (p+q)(2p-5q)$

c) $6xp - 9xq + 12yq - 8yp$
$= 3x(2p-3q) + 4y(3q-2p)$
$= 3x(2p-3q) - 4y(2p-3q)$
$= (3x-4y)(2p-3q)$

d) $2a^3 - 32a$
$= 2a(a^2 - 16)$
$= 2a(a+4)(a-4)$

e) $x^4 - 81y^4$
$= (x^2 - 9y^2)(x^2 + 9y^2)$
$= (x-3y)(x+3y)(x^2+9y^2$

f) $32y^4 - 2$
$= 2(16y^4 - 1)$
$= 2(4y^2 - 1)(4y^2 + 1)$
$= 2(2y-1)(2y+1)(4y^2+1)$

*g) $5x^2 + 5y^2 - 10xy - 45$
$= 5(x^2 + y^2 - 2xy - 9)$
$= 5[(x-y)^2 - 3^2]$
$= 5(x-y-3)(x-y+3)$

*h) $5m(m^2-4) - m^2 - 2m$
$= 5m(m+2)(m-2) - m(m+2)$
$= m(m+2)[5(m-2) - 1]$
$= m(m+2)[5m - 11]$

G) Algebraic Identities + Substitution (Intermediate)

a) If $(2a+2b)^2 = 50$ and $ab = 5$, find the value of $a^2 + b^2$
b) Given that $x^2 + y^2 = 23$ and $xy = 15$, find the value of $(3x+3y)^2$
c) Given that $x^2 - y^2 = 40$ and $x + y = 8$, find the value of $(2x-2y)^2$
***d) If $a^2 + b^2 = 218$, $ab = -91$ and $a^2 < b^2$, find the value of $a^2 - b^2$** **(advanced)**

a) $(2a+2b)^2 = 50$
$(2a)^2 + (2b)^2 + 2(2a)(2b) = 50$
$4a^2 + 4b^2 + 8ab = 50$
$4a^2 + 4b^2 + 8(5) = 50$
$4a^2 + 4b^2 = 10$
$a^2 + b^2 = 2.5$

b) $(3x+3y)^2$
$= (3x)^2 + (3y)^2 + 2(3x)(3y)$
$= 9x^2 + 9y^2 + 18xy$
$= 9(23) + 18(15)$
$= 477$

c) $x^2 - y^2 = 40$
$(x-y)(x+y) = 40$
$(x-y)(8) = 40$
$(x-y) = 5$
$(2x-2y) = 10$
$(2x-2y)^2 = 100$

*d) $(a+b)^2 = a^2 + b^2 + 2ab$
$= 218 + 2(-91)$
$= 36$
$a + b = \pm 6$

$(a-b)^2 = a^2 + b^2 - 2ab$
$= 218 - 2(-91)$
$= 400$
$a - b = \pm 20$

$a^2 - b^2 = (a+b)(a-b)$
$= \pm 6 \times \pm 20$
$= -120$ or
120 $(Rej\ since\ a^2 < b^2)$

H) Evaluate using Algebraic Identities (Intermediate)

By using a suitable algebraic identities, evaluate the following

a) 302×298
b) $35^2 + 16 - 280$
c) $3421^2 - 3420^2$
d) 97^2
e) 602^2
***f) $245^2 - 240 \times 250$** **(advanced)**

**Note: Workings and presentations are very important for these questions. No working = no marks.*

a) $302 \times 298 = (300+2)(300-2)$
$= 300^2 - 2^2$
$= 90000 - 4$
$= 89996$

$\mathbf{(a+b)(a-b) = a^2 - b^2}$

b) $35^2 + 25 - 350 = 35^2 + 5^2 - 2(35)(5)$
$= (35-5)^2$
$= 30^2 = 900$

$\mathbf{a^2 + b^2 - 2ab = (a-b)^2}$

c) $3421^2 - 3420^2 = (3421 - 3420)(3421 + 3420)$
$= (1)(6841)$
$= 6841$

$\mathbf{a^2 - b^2 = (a+b)(a-b)}$

d) $97^2 = (100-3)^2$
$= 100^2 + 3^2 - 2(100)(3)$
$= 10000 + 9 - 600$
$= 9409$

$\mathbf{(a-b)^2 = a^2 + b^2 - 2ab}$

e) $602^2 = (600+2)^2$
$= 600^2 + 2^2 + 2(2)(600)$
$= 360000 + 4 + 2400$
$= 362404$

$\mathbf{(a+b)^2 = a^2 + b^2 + 2ab}$

f) $245^2 - 240 \times 250 = 245^2 - (245-5)(245+5)$
$= 245^2 - (245^2 - 5^2)$
$= 245^2 - (245^2 - 5^2)$
$= 245^2 - 245^2 + 5^2$
$= 25$

I) Factorize Hence Find (Advanced)

a) Simplify $n^2 - (n-a)(n+a)$
b) Hence, find $1756^2 - 1752 \times 1760$

a) $n^2 - (n-a)(n+a)$
$= n^2 - (n^2 - a^2)$
$= n^2 - n^2 + a^2$
$= a^2$

b) By observation, $n = 1756$
$n + a = 1760$
$1756 + a = 1760$
$a = 1760 - 1756$
$a = 4$
since $n^2 - (n-a)(n+a) = a^2$
$\therefore 1756^2 - 1752 \times 1760 = 4^2 = 16$
Answer is 16.

Sec 2 Math: Further Expansion/Factorization

A) Expansion Example (Basic)

Simplify the following expressions:

$2(2x+y)(x-3y)-(3x-y)(2x-3y)$

B) Factorization Examples

Factorise completely

a) $4x^2y-25z^2y$

b) $2p^2-3pq-5q^2$

c) $6xp-9xq+12yq-8yp$

d) $2a^3-32a$

e) x^4-81y^4

f) $32y^4-2$

g) $5x^2+5y^2-10xy-45$

h) $5m(m^2-4)-m^2-2m$

C) Algebraic Identities + Substitution

a) If $(2a+2b)^2=50$ and $ab=5$,
find the value of a^2+b^2

b) Given that $x^2+y^2=23$ and $xy=15$,
find the value of $(3x+3y)^2$

c) Given that $x^2-y^2=40$ and $x+y=8$,
find the value of $(2x-2y)^2$

d) If $a^2+b^2=218$, $ab=-91$ and $a^2<b^2$, find the value of a^2-b^2

D) Evaluate using Algebraic Identities

By using a suitable algebraic identities, evaluate the following

a) 302×298

b) $35^2+16-280$

c) 3421^2-3420^2

d) 97^2

e) 602^2

f) $245^2-240\times250$

E) Factorize Hence Find

a) Simplify $n^2-(n-a)(n+a)$

b) Hence, find $1756^2-1752\times1760$

Quadratic Factorization (Using Calculator)

Factorise Quadratic Equations Using Calculator

Take $x^2 - x - 12$ to be the equation you want to factorise.

In your Casio fx-95SG model, calculator, press [MODE] and then [3] (EQN) and again, [3] $(ax^2 + bx + c = 0)$

For the Casio fx-97SG X model, press [MENU] and then [5] (Equation), [2] (Polynomial), [2] (select degree 2)

Insert the coefficients *a*, *b* and *c* in the respective columns. In the case of the equation given, $a = 1$, $b = -1$, and $c = -12$.

Press [=] and you will see [$X_1 = 4$]

Press [=] again and you will see [$X_2 = -3$]

Given that this calculator function is to solve quadratic equations and not factorise them, you should bring the x-value over to the left-hand side and present your factorised answer in this manner: $(x - 4)(x + 3)$

Basic Examples

1) Factorise $x^2 - 5x + 6$.

From calculator, $X_1 = 3$ *and* $X_2 = 2$

$x^2 - 5x + 6 = (x - 3)(x - 2)$

2) Factorise $5x^2 - 13x + 6$.

From calculator, $X_1 = 2$ *and* $X_2 = \frac{3}{5}$

Write $(x - 2)(x - \frac{3}{5})$

Write $(x - 2)(5x - 3)$

This should not be part of your main working, so write this at the side.

$5x^2 - 13x + 6 = (x - 2)(5x - 3)$

Trickier Examples

1) Factorise $-12x^2 - x + 1$

From calculator, $X_1 = \frac{1}{4}$ *and* $X_2 = -\frac{1}{3}$

By bringing values to the LHS, write $\left(x - \frac{1}{4}\right)\left[x - \left(-\frac{1}{3}\right)\right]$

By moving denominator up, write $(4x - 1)(3x + 1)$

This should not be part of your main working, so write this at the side.

Notice that $4x \times 3x = 12x^2$

To get $-12x^2$, we need to multiply by -1

So, your answer should be: $-12x^2 - x + 1 = -(4x - 1)(3x + 1)$

2) Factorise $2x^2 - 10x + 8$

From calculator, $X_1 = 4$ *and* $X_2 = 1$

By bringing values to the LHS, write $(x - 4)(x - 1)$

This should not be part of your main working, so write this at the side.

Notice that $x \times x = x^2$

To get $2x^2$, we need to multiply by 2

So, your answer should be: $2x^2 - 10x + 8 = 2(x - 4)(x - 1)$.

3) Factorise $x^2 - 3xy - 4y^2$

Just pretend y is not there, and type $a = 1, b = -3, c = -4$ into your calculator.

From calculator, $X_1 = 4$ *and* $X_2 = -1$

Usually, we would express the factorised equation like this: $(x - 4)(x + 1)$

But since we have an additional variable y in this question, we express it in this way: $(x - 4y)(x + y)$

*It will be helpful to always work backwards and expand your factorised form to check if it corresponds to the expression given.