(1) Simultaneous, Quadrilaterals and Inequalities

1. Find the value(s) of k for the following simultaneous equations, given that the equations have no solution.
   $(k+1)y = (2k-1)x + 5$ ---(1)
   $4y = (k+2)x + 10$ ---(2)

2. The equation $2x^2 + 8x = 1$ has roots α and β.
   a) State the value of $\alpha + \beta$ and $\alpha\beta$
   b) Find the value of $\alpha^2 - \beta^2$, leaving your answer in surd form.
   c) Find the quadratic equation whose roots are $\alpha^4 - \beta^4$

3. It is given that α and β are the roots of the equation $y = x^2 - x - 1$, where $\beta > \alpha$ and that $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ are the roots of another quadratic equation with integer coefficients. Without solving the values of α and β, find the exact value of $\alpha + \frac{1}{\alpha}$.

4. a) If one root of the equation $4x^2 - 22x + k = 0$ is ten times the other, find the value of k.
   b) Show that $2 - x^2 + 3x$ can never be greater than 5.

5. Show that the roots of the equation $x^2 + (2-k)x = \frac{3}{2}k$ are real for all real values of k.

6. The roots of the equation $x^2 - 4x + k$ differs by $2s$. Show that $s^2 = 4 - k$. Given also that the roots are positive integers and that k is a positive integer, find the possible values of s.

7. Given that α and β are the roots of the equation $x^2 = x - 5$, prove that
   a) $\frac{1-\alpha}{5} = \frac{1}{\alpha}$
   b) $\alpha^3 + 4\alpha + 5 = 0$

8. a) Find the range of values of x for which $2x^2 + x - 6$ lies between -3 and 4.
   b) Show that if the roots of the equation $2x^2 + 3x - 2 + m(x-1)^2 = 0$ are real, then m cannot be greater than $\frac{25}{12}$.

9. Find the range of values of k for which the graph of $y = kx^2 - 3x + kx$ lies entirely above the line $y = 4$.

10. i) Show that the expression $x^2 - x + \frac{7}{2}$ is always positive for all real values of x.
    ii) Hence, find the values of k which satisfy the inequality $\frac{-x^2+kx+2}{-(x^2-x+3.5)} < 2$ for all real values of x.

11. The roots of the equation $2x^2 - 8x + 50 = 0$ are α^2 and β^2. Find
    i) the value of $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$.
    ii) two different quadratic equations whose roots are α and β