Sec 2 Math: Statistics

A) Mean (Average)

The **mean** is the average of the values.

Mean = $\frac{\text{sum of data values}}{\text{number of data}}$

B) Median (Middle Number)

The **median** is the middle values that divides the data (or observations) into 2 equal parts.

The median occurs exactly at the middle position, which is the $\left(\frac{n+1}{2}\right)^{th}$ term.

C) Mode (Most Occurred Number)

The **mode** is the value that occurs most frequently in a data set.

There can be **more than 1 mode** in some distributions. The distribution will be described as Bi-modal or Tri-modal etc.

D) Table Diagram (Basic)

Five coins are thrown together 28 times. The res are shown in the table below:

No of heads shown (x)	0	1	2	3	4	5
Frequency (f)	4	4	10	8	2	0

For these results, find the
(a) mode (b) median (c) mean

(a) 2 heads
**Note: "2 heads shown" occurred 10 times. The mode is 2, not 10!*

(b) Position of median: $\frac{28+1}{2} = 14.5^{th}$ Number
Counting from left, the 14.5th Number occurs under $x = 2$.
$\therefore Median\ is\ 2.$

(c) $\frac{4\times0+4\times1+10\times2+8\times3+2\times4+0\times5}{4+4+10+8+2+0} = 2$
$\therefore Mean\ is\ 2.$

**Note: When finding mean, some students make the mistake of dividing the denominator by 6 instead of* $(4+4+10+8+2+0)$.

E) Stem and leaf diagram (basic)

Steam	Leaf
0	7 7 8
1	1 4 4 4 8
2	0 1 3 5 7 8 9 9
3	0 2 4 5 7 9
4	0 2 7

Key : 1| 1 means 11 marks

Find (i) the modal score,
(ii) the median score.
(iii) the passing mark, if 56% of pupils passed the test.

(i)The modal score is **14** marks.

(ii) There are 25 numbers.
Position of Median is $\frac{25+1}{2} = 13^{th}$ term.
Counting the 13th term from one end, the median is **27**.

(iii) 25 x 44% = 11 (11 pupils failed the test),
The 12^{th} student got the passing mark
∴ The passing marks is either **24 or 25** marks.

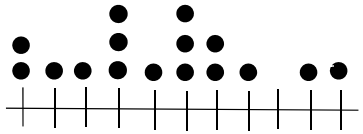
F) Mean Median and Mode (basic)

(a) The following marks are scored by 9 students in a Mathematics test marked out of a total of 15: 4,9,11,8,10,5,11,14,7.
Find
(i) the modal mark,
(ii) the median mark.
(b) The mean of nine numbers is 16. The mean of seven of these numbers is 15. Find the other two numbers if the difference between them is 3.

(a)(i) 11
(ii) **Note: Remember to arrange numbers in ascending order first!*
4, 5, 7, 8, 9, 10, 11, 11, 14
The Median (middle number) is 9.
(b) Total of 9 numbers = $16 \times 9 = 144$
Total of 7 numbers = $7 \times 15 = 105$
Total of other 2 numbers = $144 - 105 = 39$
Smaller numbers = $\frac{39-3}{2} = 18$
$18 + 3 = 21$

G) Understanding and solving dot diagrams (basic)

The following dot diagram shows the marks of 16 students in a test. The total marks for the test is 20.



(a) If the passing mark is 10 marks, find the percentage of students who failed the test.
(b) Find the (i) Mean, (ii) Median, (iii) mode.
(c) The results if another student who scored 17 marks is added to the data set. Describe the change in the mean, median and mode.

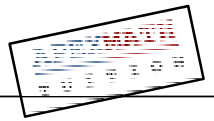
(a) $\frac{4}{16}$ x 100% = 25%
The percentage of students who failed the test is **25%**

(b)(i) $\frac{7\times2+8+9+10\times3+11+12\times3+13\times2+14+16+17}{16}$
$= 11.3125\ (marks)$

(b)(ii) Position of Median $= \frac{16+1}{2} = 8.5^{th}$ term.
$8^{th}\ term$ is 11, $9^{th}\ term$ is 12.
$\therefore 8.5^{th}$ term $= \frac{11+12}{2} = 11.5\ (marks)$

(b)(iii) **10** and **12** marks
**Note:This set of data is Bi-modal (2 modes)*

(c) $\frac{181+17}{16+1} = 11.6$
Mean = 11.6 (3.sf)
Position of Median $= \frac{17+1}{2} = 9^{th}$ term.
Median = 12
Mode = 10 and 12
No change in mode, median increased to **12** and mean increased to **11.6**



H) Given Median, find Largest/smallest x. (Important)

No. of hour	0	1	2	3
No. of students	2	x	6	15

If the median of the data is 2, find the largest and smallest possible value of x.

Solution:

No. of hour	0	1	2	3
No. of students	2	x	(1) + 5	15

To get the largest x, let the circled "1" be the median number. So its left should be equals to the right:
$2 + x = 5 + 15$
$x = 18$
The largest value of x is **18**

No. of hour	0	1	2	3
No. of students	2	x	5 + (1)	15

To get the smallest x, let the circled "1" be the median number. So its left should be equals to the right:
$2 + x + 5 = 15$
$x = 8$
The smallest value of x is **8**

I) Table Diagram (intermediate)

No. of hour	0	1	2	3	4	5
No. of students	2	3	6	15	x	5

(i) Write down the largest possible value of x, given that the mode is 3.
(ii) Write down the largest possible value of x, given that the median is 3.
(iii) Find x, given that the mean is 3.
(i) 15-1=14
The largest value of x is **14**
(ii) $2 + 3 + 6 + 14 = x + 5$
$x = 20$
The largest possible value of x is **20.**
(iii) $2 \times 0 + 3 \times 1 + 6 \times 2 + 15 \times 3 + 4x + 5 \times 5 = 85 + 4x$
$3 = \frac{85+4x}{2+3+6+15+x+5}$
$3(31 + x) = 85 + 4x$
$93 + 3x = 4x + 85$
$x = 8$

J) Intermediate Question

The mean of 9 numbers is 30. The mean of the first four numbers is 15. The last 5 numbers are the same but are different from the first 4 numbers.

(a) Find the median of the 9 numbers.
(b) If we add another number to the 9 numbers the mean of these 10 numbers is still 30, find the value of this 10^{th}number.
(c) What will happen to the median after we add the 10^{th} number from (b)? Will the median get bigger, remain the same or get smaller? Explain your reasoning.

(a) The Total of the 9 numbers:
9 x 30 = 270
The total of the first 4 numbers:
15 x 4 = 60
The total of the last 5 numbers:
270-60 = 210
Since the last 5 numbers are the same,
$\frac{210}{5} = 42$
$\frac{9+1}{2} + 5$
∴ The median is 42.
(b) $\frac{270+30}{9+1} = 30$
∴ The Value of the 10^{th} number is 30.
(c) $\frac{10+1}{2} = 5.5$
$\frac{42+30}{2} = 36$
The median will get smaller as the new median is 36.

K) Mean Median Mode Reverse (intermediate)

Dominic wrote down 5 numbers. The mean if the numbers is 7, the median is 5 and the mode is 4. If the largest number is three times the smallest number, write down the 5 numbers.
Total of Numbers:
$7 \times 5 = 35$
Since median is 5 and mode is 4,

4	4	5		

Since the largest number is 3 times the smallest number,
$4 \times 3 = 12$
$35 - 4 - 4 - 5 - 12 = 10$

4	4	5	10	12

Ans: 4,4,5,10,12

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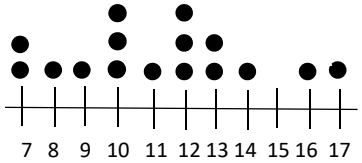

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