Sec 2 Math: Quadratic Equations

A) Quadratic Equations

An equation in the form $ax^2 + bx + c$, is known as a quadratic equation.

The constants a and b are known as the coefficient of x^2 and x respectively and c is known as the constant term.

B) Solving Quadratic Equations (Basic)

Solve each of the following equations.
a) $5x^2 - 13x + 6 = 0$
b) $3x^2 = 6x$
c) $4x^2 = 25$
d) $2x(x - 5) = 12$

a) $5x^2 - 13x + 6 = 0$
$(x - 2)(5x - 3) = 0$
$x - 2 = 0$ or $5x - 3 = 0$
$x = 2$ or $x = \frac{3}{5}$

b) $3x^2 = 6x$
$3x^2 - 6x = 0$
$3x(x - 2) = 0$
$3x = 0$ or $x - 2 = 0$
$x = 0$ or $x = 2$

c) $4x^2 = 25$
$x^2 = \frac{25}{4}$
$x = \pm\sqrt{\frac{25}{4}}$
*(*Note: Remember to put $\pm$ whenever we square root both sides of an equation)*
$x = \pm\frac{5}{2}$
$x = 2.5$ or $x = -2.5$

d) $2x(x - 5) = 12$
$2x^2 - 10x - 12 = 0$
*(*Note: For quadratic equations, always move all the terms to one side and ensure the other side is 0)*
$2(x - 6)(x + 1) = 0$
$x - 6 = 0$ or $x + 1 = 0$
$x = 6$ or $x = -1$

C) Calculator method (Optional)

When solving a quadratic equation, once we have simplified the equation into the form:
$ax^2 + bx + c = 0$
We can usually get the final answer using our calculators.

For Casio fx-97SGX, press "Menu", "5", "2", "2". Then fill in the values of a, b and c and press "=" to get the answers.
Note: This is only for checking answers, you still need to show presentation!
For other calculators, please ask your tutor/teacher.

D) Common Errors (Refer to previous box for the correct method.)

Solve each of the following equations.
a) $3x^2 = 6x$
b) $4x^2 = 25$
c) $2x(x - 5) = 12$

a) $3x^2 = 6x$

$3x = 6$ ↻✗ (Cancel x from both sides)
Common Error: The above step is WRONG! When we cancel x from both sides, we cancelled away one possible answer. We should factorize x out instead!!!

b) $4x^2 = 25$
$x^2 = \frac{25}{4}$
$x = \sqrt{\frac{25}{4}}$ ↻✗
$x = 2.5$
Common Error: The above step is WRONG! Don't forget to put "$\pm$"whenever we square root both sides of an equation!!!

c) $2x(x - 5) = 12$
$2x = 12$ or $x - 5 = 12$ ↻✗
Common Error: The above step is WRONG! Only when equals 0, then we can split it up using "zero product rule"!!!

E) Given Solution, find Equation (Intermediate)

If $x = 3$ is a solution of the equation
$2x^2 + kx - 15 = 0$, find
a) the value of k,
b) the other solution of the equation.

a) Sub $x = 3$ into the equation:
$2(3)^2 + k(3) - 15 = 0$
$3 + 3k = 0$
$k = -1$

b) Sub $k = -1$ into initial equation:
$2x^2 + (-1)x - 15 = 0$
$(2x + 5)(x - 3) = 0$
$2x + 5 = 0$ or $x - 3 = 0$
$x = -2.5$ or $x = 3$ (Not applicable)

F) "Hence, Solve" Questions (Intermediate)

a) Factorize $3n^2 - 13n - 10$
b) Hence, solve
** i) $3(x + 1)^2 - 13(x + 1) - 10 = 0$**
** ii) $3(2y)^2 - 13(2y) = 10$**

a) $3n^2 - 13n - 10$
$= (n - 5)(3n + 2)$

Factorize using cross box or multiplication frame

bi) Substitute $n = x + 1$ into above expression:
$3(x + 1)^2 - 13(x + 1) - 10 = 0$
$((x + 1) - 5)(3(x + 1) + 2) = 0$
$(x - 4)(3x + 5) = 0$
$x - 4 = 0$ or $3x + 5 = 0$
$x = 4$ or $x = -\frac{5}{3}$

bi) Substitute $n = 2y$ into above expression:
$3(2y)^2 - 13(2y) = 10$
$3(2y)^2 - 13(2y) - 10 = 0$
$((2y) - 5)(3(2y) + 2) = 0$
$(2y - 5)(6y + 2) = 0$
$2y - 5 = 0$ or $6y + 2 = 0$
$y = \frac{5}{2}$ or $y = -\frac{1}{3}$

Note: Part (b) is a "hence" question, therefore we are obliged to make use of part (a) answer to solve the question. The key is to identify the "substitution" involved and make use of it properly.

G) Solving Equations (Intermediate)

Given that $25x^2 - 10xy + y^2 = 0$, find the value of $\frac{10x}{y}$

$25x^2 - 10xy + y^2 = 0$
$(5x - y)^2 = 0$
$5x - y = 0$
$5x = y$
$\frac{5x}{y} = 1$
$\frac{x}{y} = \frac{1}{5}$

$\therefore \frac{10x}{y} = 10 \times \frac{1}{5}$
$= 2$

H) Word Problem (Intermediate)

Danial walks for $2x$ hours at a speed of $(x - 4)$ km/h and cycles for $(2x - 1)$ hours at a speed of $(x + 5)$ km/h.
a) Express the distance Danial has walked in terms of x
b) Express the distance Danial has cycled in terms of x.
c) The total distance is 90 km. Form an equation in x and show that $4x^2 + x - 95 = 0$
d) Hence, find the time taken by Danial for walking.

a) $2x \times (x - 4)$
$= 2x^2 - 8x$ km

b) $(2x - 1)(x + 5)$
$= 2x^2 + 9x - 5$ km

c) $(2x^2 - 8x) + (2x^2 + 9x - 5) = 90$
$4x^2 + x - 5 = 90$
$4x^2 + x - 95 = 0$

d) $4x^2 + x - 95 = 0$
$(4x - 19)(x + 5) = 0$
$4x - 19 = 0$ or $x + 5 = 0$
$x = \frac{19}{4}$ or $x = -5$

e) $x = \frac{19}{4}$ since time has to be a positive value,
Danial walked for $2\left(\frac{19}{4}\right) = 9.5$ hours

APEXMATH TUITION

Quadratic Equation + Algebraic Fractions

I) Quadratic Equations involving Algebraic Fractions

Solve each of the following equations.

a) $\frac{3p}{2+p^2} = \frac{1}{3-2p}$

b) $\frac{x-3}{2} - \frac{4}{x+4} = 0$

c) $\frac{8}{x} - \frac{5}{x+1} = 1$

d) $4 - 4x + \frac{3}{x} = 0$

a) $\frac{3p}{2+p^2} = \frac{1}{3-2p}$

$3p(3-2p) = 1(2+p^2)$ (Cross multiply)

$9p - 6p^2 = 2 + p^2$

$7p^2 - 9p + 2 = 0$

$(7p-2)(p-1) = 0$

$7p - 2 = 0$ or $p - 1 = 0$

$p = \frac{2}{7}$ or $p = 1$

b) $\frac{x-3}{2} - \frac{4}{x+4} = 0$

$\frac{x-3}{2} = \frac{4}{x+4}$

$(x-3)(x+4) = 8$

$x^2 + x - 12 = 8$

$x^2 + x - 20 = 0$

$(x+5)(x-4) = 0$

$x + 5 = 0$ or $x - 4 = 0$

$x = -5$ or $x = 4$

c) $\frac{8}{x} - \frac{5}{x+1} = 1$

$\frac{8(x+1)-5(x)}{x(x+1)} = 1$

*(*Note: Combine fractions by making same denominator)*

$8(x+1) - 5(x) = x(x+1)$

$8x + 8 - 5x = x^2 + x$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

d) $4 - 4x + \frac{3}{x} = 0$

$\frac{4x-4x^2+3}{x} = \frac{0}{1}$

$4x - 4x^2 + 3 = 0(x)$ (Cross Multiply)

$4x - 4x^2 + 3 = 0$ (*Note: $0 \times x = 0$)

$(2x-3)(2x+1) = 0$

$x = \frac{3}{2}$ or $x = -\frac{1}{2}$

APEX**MATH** TUITION

J) Word Problems (Speed-Time-Distance)

Mr Tay makes regular business trips to Malacca which is 240 km from Singapore.

(a) On the journey to Malacca, he travels at an average speed of x km/h. Write down an expression in x for the time taken in hours to travel from Singapore to Malacca.

(b) On his return journey to Singapore, he increases his speed by 10 km/h. Write down an expression in terms of x for the time taken in hours to travel from Malacca to Singapore.

(c) If the difference between the two journeys is 20 minutes, form an equation in x and show that it reduces to $x^2 + 10x - 7200 = 0$.

(d) Solve the equation $x^2 + 10x - 7200 = 0$.

(e) Find the total time taken for the trip from Singapore to Malacca.

a) Time taken to travel from SG to Malacca = $\frac{240}{x} h$

b) Time taken to travel from Malacca to SG = $\frac{240}{x+10} h$

c) $\frac{240}{x} - \frac{240}{x+10} = \frac{20}{60}$ *(*Note: It is $\frac{20}{60}$ not 20 because in hours)*

$\frac{240(x+10)-240(x)}{x(x+10)} = \frac{1}{3}$

$\frac{240x+2400-240x}{x^2+10x} = \frac{1}{3}$

$3(2400) = x^2 + 10x$

$x^2 + 10x - 7200 = 0$ (shown)

d) $x^2 + 10x - 7200 = 0$

$(x-80)(x+90) = 0$

$x = 80 \ or \ x = -90$

*(*Note: We do not reject -90 in this part because the question in this part plainly asks us to solve the equation and it is not with respect to the scenario)*

e) $x = 80$

$(x = -90 \ is \ rejected \ since \ speed \ must \ be \ positive)$

Total time taken from SG to Malacca = $\frac{240}{(80)} = 3h$

K) Word Problems (Pipe Rate)

An aquarium tank can be filled by two taps A and B in 4 hours. Tap A can fill up the tank in x hours while Tap B takes $(3x-6)$ hours to fill.

(a) Find the fraction of the tank that can be filled up in 1 hour by
(i) Tap A **(ii)** Tap B

(b) Form an equation in x and show that it reduces to $3x^2 - 22x + 24 = 0$.

(c) Solve the equation $3x^2 - 22x + 24 = 0$.

(d) Explain why one of the solutions in (c) is not applicable.

ai) Tap A in 1 hour = $\frac{1}{x} \ of \ tank$

aii) Tap B in 1 hour = $\frac{1}{3x-6} \ of \ tank$

b) Together, $\frac{1}{4}$ of tank will be filled in 1 hour.

$\frac{1}{x} + \frac{1}{3x-6} = \frac{1}{4}$

$\frac{(3x-6)+x}{x(3x-6)} = \frac{1}{4}$

$\frac{4x-6}{3x^2-6x} = \frac{1}{4}$

$16x - 24 = 3x^2 - 6x$

$3x^2 - 22x + 24 = 0$ (shown)

c) $(3x-4)(x-6) = 0$

$3x - 4 = 0$ or $x - 6 = 0$

$x = \frac{4}{3}$ or $x = 6$

d) $x = \frac{4}{3}$ is not applicable because x has to be greater than 4 hours. Tap A alone cannot fill faster than when both Tap A and Tap B are filling together.

L) Word Problems ($/Profit)

Mrs Ong bought a crate of mangoes for $300. She paid $$x$ for each mango.

(a) Write down an expression for the number of mangoes she bought.

(b) As 10 of the mangoes were rotten, she decided to sell each of the remaining mangoes at a profit of $0.50. Write down an expression in terms of x for the sum of money she would receive if all the remaining mangoes were sold.

(c) From the sale of all the remaining mangoes, Mrs Ong made a profit of $30. Write down an equation in x to represent this information, and show that it reduces to $2x^2 + 7x - 30 = 0$.

(d) Solve the equation and find the cost price of each mango.

a) Number of mangoes bought = $\frac{300}{x}$

b) Remaining mangoes = $\frac{300}{x} - 10$

Money made= $(x + 0.5)(\frac{300}{x} - 10)$

$= 300 - 10x + \frac{150}{x} - 5$

$= 295 - 10x + \frac{150}{x}$

c) $\left(295 - 10x + \frac{150}{x}\right) = 300 + 30$

$-35 - 10x + \frac{150}{x} = 0$

$-35x - 10x^2 + 150 = 0$

$10x^2 + 35x - 150 = 0$

$2x^2 + 7x - 30 = 0$ (shown)

d) $2x^2 + 7x - 30 = 0$

$(2x-5)(x+6) = 0$

$x = \frac{5}{2} \ or \ x = -6 \ (rejected \ since \ x > 0)$

Cost price of each mango = $2.50

APEX**MATH**
T U I T I O N

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(a) Find the fraction of the tank that can be filled up in 1 hour by

  (i) Tap A

  (ii) Tap B

(b) Form an equation in x and show that it reduces to $2x^2 - 6x - 9 = 0$.

(c) Solve the equation $2x^2 - 6x - 9 = 0$, giving your answers correct to two decimal places.

(d) Explain why one of the solutions in (c) is rejected.

(e) If Tap B alone is used, find the amount of extra time taken, to the nearest minute, for it to fill the tank.

L) Word Problem (% and Profit)

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