Sec 1 Math: Percentage

A) Introduction to percentage

Percentage in the value of a quantity can either be expressed as a percentage increase or decrease in the original value.

Increase/Decrease = $\frac{Increase/decrease}{Original\ value} \times 100\%$

Profit is calculated as the selling price minus the cost price. Whereas Loss is calculated as the cost price minus the selling price.

Profit = Selling price – Cost price
Loss = Cost price – Selling price

Discounts are calculated as the marked price (original price) minus the sale price (cheaper price)

The formula to calculate percentage discount is:
% Discount = $\frac{Discount}{Marked\ price} \times 100\%$

GST or goods and services tax is a tax on consumption. This tax is applicable to purchases on good or services.

B) Worded percentage question (Basic)

In a Secondary 1 cohort of a co-ed school, 55% of the student population are boys. (a) Given that there are 135 girls, find the total enrolment of the Secondary 1 cohort. (b) If there is 1 teacher for every 5 boys there are, find the number of teachers.

(a) Percentage of girls:
100% - 55%
= 45%
The total enrolment of the Secondary 1 cohort:
$135 \div \left(\frac{45}{100}\right)$
= 300
(b) Number of boys:
$=135 \div \left(\frac{45}{55}\right)$
= 165
Total amount of teachers:
165 ÷ 5
= 33

C) Finding percentage of marks (Basic)

The highest number of marks obtainable at a Math competition is 60. Kate obtains 40 marks, Priya obtains 46 marks and Nora obtains 49 marks. The exam board decides that those who score 80% and above get a gold award, those who score 70% to 79% inclusive get a silver award and those who score 60% to 69% inclusive get a bronze award. Determine the type of award each girl gets.

Percentage of Kate's marks:
$\frac{40}{60} \times 100\%$
= 66.7% (3 s.f.)
Percentage of Priya's marks:
$\frac{46}{60} \times 100\%$
= 76.7% (3 s.f.)
Percentage of Nora's marks:
$\frac{49}{60} \times 100\%$
= 81.7% (3 s.f.)
∴ Kate gets the **bronze award,** Priya gets the **silver award** and Nora gets **the gold award.**

D) Finding percentage of marks (Basic)

Ethan's monthly salary is $1850. In a particular month, he spent 20.5% of his salary on room rental, $690 on food and $940 on other expenses. Express the amount he overspent as a percentage of his monthly salary. (correct to 2 decimal places.)

The amount of money he spent on room rental:
1850 × 20.5%
= $379.25
Total amount of money spent in that month:
379.25 + 940 + 690
= $2009.25
The amount of money he overspent:
2009.25 – 1850
= $159.25
The percentage of his monthly salary he overspent:
$\frac{159.25}{1850} \times 100\%$
= 8.61% (2 decimal places)

E) GST/Service Charge (Basic)

Natalie orders one bowl of ramen at a Japanese restaurant which offers an 18% discount. There is a service charge of 10% and GST is at 7%. Given that she pays a total of $10.13, find the marked price of the ramen.

Price before GST $= \$10.13 \div 1.07$
$= \$9.4672897$

Price before Service charge
$= \$9.4672897 \div 1.10$
$= \$8.60662702$

Price before Discount
$= \$8.60662702 \div 0.82$
$= \$10.50$ (Nearest Cent)

Note: GST is usually added last and 10% is added on top of Service charge.

F) Down payment with unknown interest (Basic)

Mr Goh bought a home entertainment system on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was $8000. (a) Calculate the selling price of the system. (b) Mr Goh paid the balance of payment in monthly instalments of $380, charged at x% per annum simple interest for 2 years. Find the value of x.

(a) $8000 \div \left(\frac{100-20}{100}\right)$
= $10 000

(b) Total Sum of all monthly instalments $= 380 \times (12 \times 2)$
$= \$9120$
Total interest paid $= 9120 - 8000$
$= \$1120$
$I = \frac{PRT}{100}$ (Formula for simple interest)
$1120 = \frac{8000 \times x \times 2}{100}$
$112\ 000 = 8000 \times x \times 2$
$x = 7$

G) Cost price, Selling price and marked price (Basic)

A man bought a watch for $560. He made a profit of 15% of the cost price after selling the watch at a discount of 20% of the marked price. Find the marked price of the watch.

Actual selling price of watch:
$= \$560 \times \frac{115}{100}$
= $644
The actual marked price of the watch:
$= \$644 \times \frac{100}{80}$
= $805

H) Percentage involving down payments and interest (Basic)

The cash price of a sofa was $4000. Mr Chou paid a down payment and arranged to pay the remaining amount by monthly instalments. Simple interest of 4% per annum was charged and made monthly instalments of $260 for one year. Find the amount he paid as a down payment.

Total sum of monthly instalments:
$= \$260 \times 12$
$= \$3120$
$3120 represents 104%
Principal borrowed $= 3120 \times \frac{100}{104}$
$= \$3000$
Downpayment $= 4000 - 3000$
$= \$1000$

I) Income and commission (Basic)

Joey's monthly income consists of basic salary of $500 and a commission of 4% on her sales for the month. If her income is $1220 for a particular month, find her sales for that month.

Money made from commission:
1220 – 500
= $720
Her sales that month:
$720 \times \frac{100}{4}$
=$18 000

J) Cost price and selling price (Basic)

Cup A is 40% filled with water. Cup B, which is identical to Cup A is completely filled with a mixed solution containing 70% water and 30% hydrochloric acid. 60% of content in Cup B is then poured into Cup A. After mixing, 60% of the mixed solution in Cup A is poured into Cup B. Find the percentage of water in Cup A now.

Percentage of water poured from B to A:
70 × 60%
= 42
Percentage of hydrochloric acid poured from B to A:
30 × 60%
= 18
Percentage of water poured from A to B:
(42 + 40) × 60%
= 49.2
Percentage of water left in A:
82 – 49.2
= 32.8%

K) Percentage change involving algebra (Intermediate)

A rectangle has its length increased by 20% and its width decreased by 20%. Find, if any, the percentage change in its area.

Let x be the original length.
New length:
$120\% \times x$
$= 1.2x$
Let y be the original width
$80\% \times x$
$= 0.8y$
New area of the rectangle:
$1.2x \times 0.8y$
$= 0.96xy$
Percentage change:
$\frac{0.96xy - xy}{xy} \times 100\%$
$= -\frac{0.04xy}{xy} \times 100\%$
$= -4\%$
∴ The percentage change in its area is -4%

A) Worded percentage question (Basic)

In a Secondary 1 cohort of a co-ed school, 55% of the student population are boys.
(a) Given that there are 135 girls, find the total enrolment of the Secondary 1 cohort.
(b) If there is 1 teacher for every 5 boys there are, find the number of teachers.

B) Finding percentage of marks (Basic)

The highest number of marks obtainable at a Math competition is 60.
Kate obtains 40 marks, Priya obtains 46 marks and Nora obtains 49 marks.
The exam board decides that those who score 80% and above get a gold award,
those who score 70% to 79% inclusive get a silver award
and those who score 60% to 69% inclusive get a bronze award.
Determine the type of award each girl gets.

C) Finding percentage of marks (Basic)

Ethan's monthly salary is $1850. In a particular month, he spent 20.5% of his salary on room rental,
$690 on food and $940 on other expenses.
Express the amount he overspent as a percentage of his monthly salary. (correct to 2 decimal places.)

D) Percentage involving down payments and interest (Basic)

Natalie orders one bowl of ramen at a Japanese restaurant which offers an 18% discount.
There is a service charge of 10% and GST is at 7%.
Given that she pays a total of $10.13, find the marked price of the ramen

E) Down payment with unknown interest (Basic)

Mr Goh bought a home entertainment system on hire purchase.
He paid a deposit of 20% of the selling price. The balance of the payment was $8000.
(a) Calculate the selling price of the system.
(b) Mr Goh paid the balance of payment in monthly instalments of $380,
charged at x% per annum simple interest for 2 years. Find the value of x

F) Finding percentage of marks (Basic)

A man bought a watch for $560. He made a profit of 15% of the cost price
after selling the watch at a discount of 20% of the selling price. Find the actual selling price of the watch.

G) Percentage involving down payments and interest (Basic)

The cash price of a sofa was $4000. Mr Chou paid a down payment and
arranged to pay the remaining amount by monthly instalments.
Simple interest of 4% per annum was charged and made monthly instalments of $260 for one year.
Find the amount he paid as a down payment.

H) Income and commission (Basic)

Joey's monthly income consists of basic salary of $500 and a commission of 4% on her sales for the month.
If her income is $1220 for a particular month, find her sales for that month.

I) Cost price and selling price (Basic)

Cup A is 40% filled with water. Cup B, which is identical to Cup A is completely filled with a mixed solution containing 70% water and 30% hydrochloric acid. 60% of content in Cup B is then poured into Cup A. After mixing, 60% of the mixed solution in Cup A is poured into Cup B. Find the percentage of water in Cup A now.

J) Percentage change involving algebra (Intermediate)

A rectangle has its length increased by 20% and its width decreased by 20%.
Find, if any, the percentage change in its area.