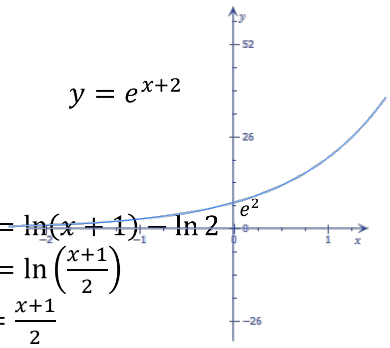
Modulus and Graph of Logarithms/Exponentials Functions

1 Sketch the graph $y = e^{x+2}$. State the equation of a straight line that can be drawn to solve the equation $x + 2 = \ln(x + 1) - \ln 2$.



$x + 2 = \ln(x + 1) - \ln 2$

$x + 2 = \ln\left(\frac{x+1}{2}\right)$

$e^{x+2} = \frac{x+1}{2}$

A suitable straight line is $y = \frac{x+1}{2}$

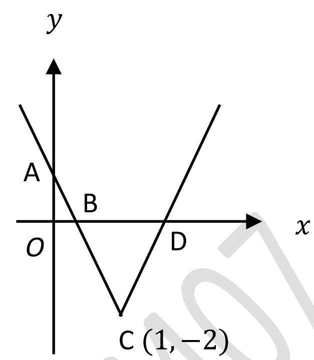
2 The figure shows part of the graph of $y = |ax - 4| + b$ where $C(1, -2)$ is the minimum point of the graph.

i) State the value of b

ii) Find the value of a

iii) Find the coordinates of A, B and D.

iv) Write down the range of values of x for which y is negative.



i) $b = -2$

ii) Sub $(1, -2)$ into equation:

$-2 = |a(1) - 4| - 2$

$a - 4 = 0$

$a = 4$

iii) When $x = 0$,

$y = |0 - 4| - 2 = 2$

Coordinates of A is $(0,2)$

When $y = 0$,

$0 = |4x - 4| - 2$

$0 = (4x - 4) - 2$ or $0 = (-4x + 4) - 2$

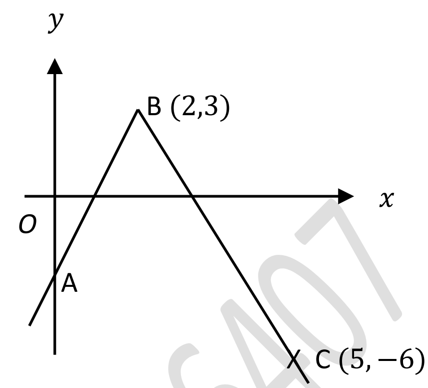
$4x = 6$ or $4x = 2$

$x = 1.5$ or $x = 0.5$

Coordinates of B is $(0.5,0)$ and Coordinates of D is $(1.5,0)$

iv) $0.5 < x < 1.5$

3 The diagram show part of the graph of $y = a - |bx + c|$ where $b > 0$. Given that it passes through the points $B(2,3)$ and $C(5,-6)$,
i) Find the values of a, b and c
ii) Find the x-intercepts and the y-intercept of the graph.



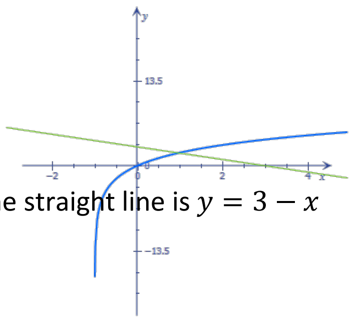
i) By observation, $a = 3$.
Gradient of line BC $= \frac{-6-3}{5-2}$
$= -3$
Since $b > 0$, $b = 3$
Sub $(2,3)$ into equation:
$3 = 3 - |3(2) + c|$
$c = -6$
$\therefore$ Equation is $y = 3 - |3x - 6|$

ii) When $x = 0$,
$y = 3 - |0 - 6| = -3$
y-intercept is $(0, -3)$

When $y = 0$,
$0 = 3 - |3x - 6|$
$3x - 6 = 3$ or $3x - 6 = -3$
$x = 3$ or $x = 1$
x-intercepts are $(1,0)$ and $(3,0)$.

4 Sketch the graph $y = 3\ln(x+1)$. On the same graph, add a suitable straight line which will help solve the equation $(x+1)e^{\frac{1}{3}x+1} = e^2$. State the equation of the straight line.

$(x+1)e^{\frac{1}{3}x+1} = e^2$
$(x+1) = e^{1-\frac{1}{3}x}$
$\ln(x+1) = 1 - \frac{1}{3}x$
$3\ln(x+1) = 3 - x$



The straight line is $y = 3 - x$

5 Solve the equation $|-3x+21| = 8x + |x-7|$

$|-3x+21| = 8x + |x-7|$
$|-3(x-7)| = 8x + |x-7|$
$|-3||x-7| - |x-7| = 8x$
$|x-7|(3-1) = 8x$
$x - 7 = 4x$ or $x - 7 = -4x$
$x = -\frac{7}{3}$ or $x = \frac{7}{5}$

Check by Substitution:

$x = -\frac{7}{3}$ or $x = \frac{7}{5}$

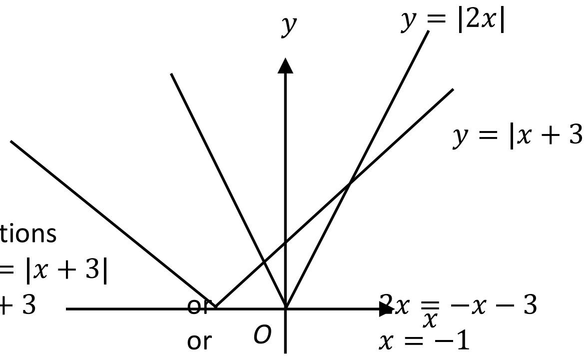
$LHS = \left|-3(-\frac{7}{3}) + 21\right| = 28$ or $LHS = \left|-3(\frac{7}{5}) + 21\right| = \frac{84}{5}$

$RHS = 8\left(-\frac{7}{3}\right) + \left|\left(-\frac{7}{3}\right) - 7\right| = -\frac{28}{3}$ or $RHS = 8\left(\frac{7}{5}\right) + \left|\left(\frac{7}{5}\right) - 7\right| = \frac{84}{5}$

$\boldsymbol{LHS \neq RHS, \therefore x = -\frac{7}{3} \text{ is rejected}}$ $\boldsymbol{LHS = RHS, \therefore x = \frac{84}{5}}$

6 i) On the same diagram, sketch the graphs of $y = |2x|$ and $y = |x + 3|$.
ii) State the number of solutions of the equation for $|2x| = |x + 3|$.
iii) Find the coordinates of the intersection points of the 2 graphs.
iv) Hence, state the solution of $|2x| > |x + 3|$.

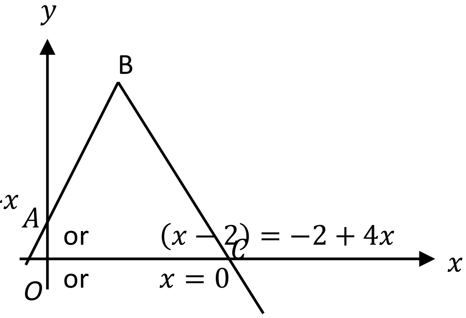
i)



ii) 2 solutions
iii) $|2x| = |x + 3|$
$2x = x + 3$ or $2x = -x - 3$
$x = 3$ or $x = -1$
$y = 6$ or $y = 2$
Coordinates of intersection points are $(-1,2)$ and $(3,6)$
iv) $x < -1$ or $x > 3$

7 a) Solve the equation $|x-2| = 2-4x$

b) The diagram shows part of the graph of $y = 4 - |2x-3|$. Find the coordinates of A, B and C.



a) $|x-2| = 2-4x$

$(x-2) = 2-4x$ or $(x-2) = -2+4x$

$x = \frac{4}{5}$ (Rej) or $x = 0$

Substitute $x = \frac{4}{5}$ back into equation:

$\left|\frac{4}{5} - 2\right| = 2 - 4\left(\frac{4}{5}\right)$

$\frac{6}{5} \neq -\frac{6}{5}$

$\therefore x = \frac{4}{5}$ is rejected

Substitute $x = 0$ back into equation:

$|0-2| = 2-4(0)$

$2 = 2$

$\therefore x = 0$

b) When $x = 0$,

$y = 4 - |2(0) - 3|$

$= 1$

$\therefore A(0,1)$

When $y = 0$,

$0 = 4 - |2x-3|$

$2x - 3 = 4$ or $2x - 3 = -4$

$x = 3.5$ or $x = -0.5$

$\therefore C(3.5, 0)$

y-coordinate of $B = 4$

When $y = 4$,

$4 = 4 - |2x-3|$

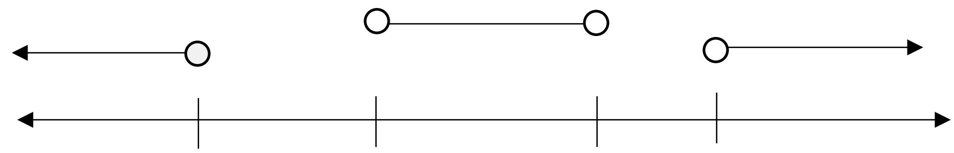
$2x - 3 = 0$

$x = 1.5$

$\therefore B(1.5, 4)$

8 Solve the inequality $|2x^2 + 4x - 11| > 5$

$(2x^2 + 4x - 11) > 5$	or	$-(2x^2 + 4x - 11) > 5$
$2x^2 + 4x - 11 > 5$	or	$2x^2 + 4x - 11 < -5$
$2x^2 + 4x - 16 > 0$	or	$2x^2 + 4x - 6 < 0$
$(x + 4)(x - 2) > 0$	or	$(x + 3)(x - 1) < 0$
$x < -4$ or $x > 2$	or	$-3 < x < 1$



$\therefore x < -4$ or $-3 < x < 1$ or $x > 2$

9 i) Sketch the graph $y = |x^2 - 2x|$indicating the intercepts and coordinates of the turning point.
ii) In each of the following case, determine the number of solutions of the equation $|x^2 - 2x| = mx + c$ where $0 < c < 1$, justify your answer.

a) $m = 0$

b) $m = -1$

i) Plot $y = x^2 - 2x$ first, then reflect portions of graph below x-axis on the x-axis.

$y = x(x - 2)$

x-intercepts are at $(0,0)$ and $(2,0)$.

Since quadratic graph is symmetrical, turning point is at $x = 1$.

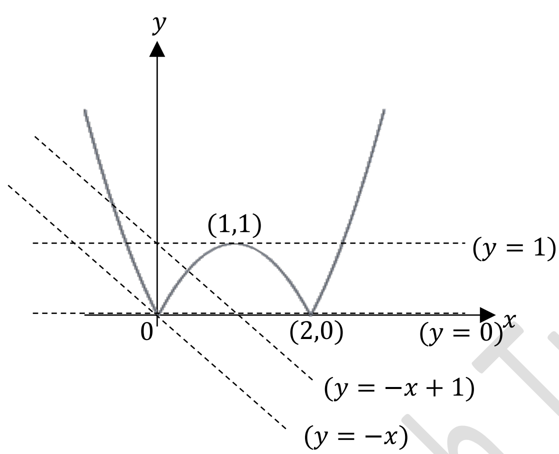
At $x = 1$, $y = 1(1 - 2) = -1$

Turning point is $(1, -1)$

Sub $x = 0$,

$y = 0$

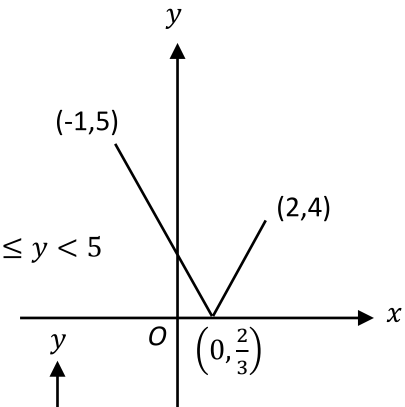
y-intercept is $(0,0)$.



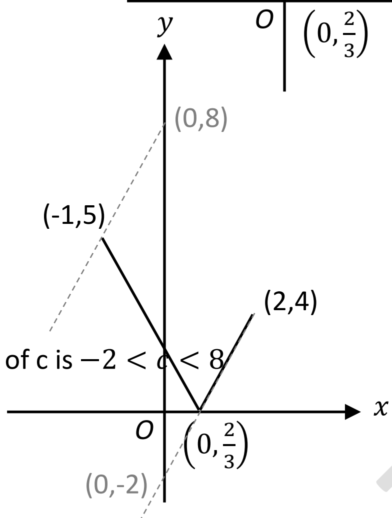
iia) There are 4 solutions since the graphs cut at 4 points.

iib) There are 2 solutions since the graphs cut at 2 points.

10 i) Sketch the graph of $y = |3x - 2|$ for $-1 < x < 2$.
ii) State the corresponding range of y.
iii) Find the range of values of c for which $|3x - 2| = 3x + c$ has only one solution for $-1 < x < 3$.



Range of y is $0 \leq y < 5$



Range of c is $-2 < c < 8$