Sec 2 Math: Algebraic Fractions and Making Subject

A) Algebraic Fractions

Fractions that contain algebraic variables in either the numerator, denominator or both.

B) Common mistake

Simplify $\frac{2a+y}{3a+5}$

$\frac{2\cancel{a}+y}{3\cancel{a}+5} = \frac{2+y}{3+5}$ **(NOT ALLOWED!)**

We are NOT ALLOWED to cancel a from top and bottom in this case because both the top and the bottom are made up of 2 terms each with a "plus sign" in between the terms

C) Single Fraction without "Plus and Minus"

Simplify $\frac{(2a^3b)^2}{8ab^5}$

$\frac{(2a^3b)^2}{8ab^5} = \frac{4a^6b^2}{8ab^5}$

$= \frac{\cancel{4}a^{\cancel{6}5}\cancel{b^2}}{\cancel{8}2\cancel{a}b^{\cancel{5}3}}$

$= \frac{a^5}{2b^3}$

D) General Rule when you see Algebraic fractions:

Whenever you see an algebraic fraction, First step is always to FACTORISE!

Look at the examples in the next few boxes to understand.

E) Algebraic Fractions (Single Fraction)

Simplify each of the following.

a) $\frac{3x}{6x^2+5x}$

b) $\frac{6m^2n-9mn^2}{2m^2-3mn}$

c) $\frac{18x^2-50}{6x^2-x-15}$

d) $\frac{2a^2-ac-c^2}{a^2+ab-ac-bc}$

***e)** $\frac{(3x+2y)^2-6x-4y}{9x^2-4y^2}$ **(Advanced)**

First step of Algebraic fraction is to Always Factorize when Possible

a) $\frac{3x}{6x^2+5x} = \frac{3x}{x(6x+5)}$

$= \frac{3}{6x+5}$

b) $\frac{6m^2n-9mn^2}{2m^2-3mn} = \frac{3mn\cancel{(2m-3n)}}{\cancel{m(2m-3n)}}$

$= 3n$

c) $\frac{18x^2-50}{6x^2-x-15} = \frac{2(9x^2-25)}{(2x+3)(3x-5)}$

$= \frac{2\cancel{(3x-5)}(3x+5)}{(2x+3)\cancel{(3x-5)}}$

$= \frac{2(3x+5)}{2x+3}$

d) $\frac{2a^2-ac-c^2}{a^2+ab-ac-bc} = \frac{(2a+c)(a-c)}{a(a+b)-c(a+b)}$

$= \frac{(2a+c)\cancel{(a-c)}}{\cancel{(a-c)}(a+b)}$

$= \frac{2a+c}{a+b}$

e) $\frac{(3x+2y)^2-6x-4y}{9x^2-4y^2} = \frac{(3x+2y)^2-2(3x+2y)}{(3x+2y)(3x-2y)}$

$= \frac{\cancel{(3x+2y)}[(3x+2y)-2]}{\cancel{(3x+2y)}(3x-2y)}$

$= \frac{3x+2y-2}{3x-2y}$

F) Multiplication and Division of Algebraic Fractions

Simplify each of the following.

a) $\frac{5a^2b^2}{18c} \times \frac{2b^2c^3}{15a(2ab^2)}$

b) $\frac{y^2-y-2}{2-6y} \div \frac{3y^2-12}{2y+4} \times \frac{1}{y+1}$

c) $\frac{2y^2-3y-5}{y^2-1} \times \frac{(y-1)^2}{y-1}$

***d)** $\left(n-\frac{9}{n}\right) \div \left(11-2n-\frac{15}{n}\right)$

(Advanced)

a) $\frac{5a^2b^2}{18c} \times \frac{2b^2c^3}{15a(2ab^2)} = \frac{10a^2b^4c^3}{540ca^2b^2}$

$= \frac{b^2c^2}{54}$

b) $\frac{y^2-y-2}{2-6y} \div \frac{3y^2-12}{2y+4} \times \frac{1}{y+1}$

$= \frac{(y-2)(y+1)}{2(1-3y)} \times \frac{2(y+2)}{3(y^2-4)} \times \frac{1}{y+1}$

$= \frac{\cancel{(y-2)}\cancel{(y+1)}}{\cancel{2}(1-3y)} \times \frac{\cancel{2}\cancel{(y+2)}}{3\cancel{(y-2)}\cancel{(y+2)}} \times \frac{1}{\cancel{y+1}}$

$= \frac{1}{3(1-3y)}$

*Top can cancel with bottom

c) $\frac{2y^2-3y-5}{y^2-1} \times \frac{(y-1)^2}{y-1}$

$= \frac{(2y-5)\cancel{(y+1)}}{\cancel{(y-1)}\cancel{(y+1)}} \times \frac{\cancel{(y-1)^2}}{\cancel{y-1}}$

$= 2y-5$

d) $\left(n-\frac{9}{n}\right) \div \left(11-2n-\frac{15}{n}\right)$

$= \left(\frac{n^2-9}{n}\right) \div \left(\frac{11n-2n^2-15}{n}\right)$

$= \left(\frac{n^2-9}{n}\right) \times \left(\frac{n}{11n-2n^2-15}\right)$

$= \frac{\cancel{(n-3)}(n+3)}{\cancel{n}} \times \frac{\cancel{n}}{(-2n+5)\cancel{(n-3)}}$

$= \frac{n+3}{-2n+5}$

G) Addition and Subtraction of Algebraic Fractions

Simplify each of the following.

a) $2-\frac{3n+4}{12n}-\frac{5}{4n}$

b) $\frac{4}{x-3}+\frac{6}{x+3}$

c) $\frac{-18y}{y^2-3y-18}-\frac{2y}{y+3}$

d) $\frac{3x}{x^2-16}-\frac{2}{x+4}-\frac{3}{8-2x}$

When add/subtract algebraic fractions, Please make denominator the same!

and… Always factorise first

a) $2-\frac{3n+4}{12n}-\frac{5}{4n}$

$= \frac{2(12n)}{12n}-\frac{3n+4}{12n}-\frac{15}{12n}$

$= \frac{24n-3n-4-15}{12n}$

$= \frac{21n-19}{12n}$

b) $\frac{4}{x-3}+\frac{6}{x+3}$

$= \frac{4(x+3)}{(x-3)(x+3)}+\frac{6(x-3)}{(x+3)(x-3)}$

$= \frac{4(x+3)+6(x-3)}{(x-3)(x+3)}$

$= \frac{4x+12+6x-18}{(x-3)(x+3)}$

$= \frac{10x-6}{(x-3)(x+3)}$

c) $\frac{-18y}{y^2-3y-18}-\frac{2y}{y+3}$

$= \frac{-18y}{(y+3)(y-6)}-\frac{2y}{y+3}$

$= \frac{-18y-2y(y-6)}{(y+3)(y-6)}$

$= \frac{-18y-2y^2+12y}{(y+3)(y-6)}$

$= \frac{-6y-2y^2}{(y+3)(y-6)}$

$= \frac{-2y\cancel{(3+y)}}{\cancel{(y+3)}(y-6)}$

$= \frac{2y}{6-y}$

d) $\frac{3x}{x^2-16}-\frac{2}{x+4}-\frac{3}{8-2x}$

$= \frac{3x}{(x+4)(x-4)}-\frac{2}{x+4}-\frac{3}{2(4-x)}$

$= \frac{3x}{(x+4)(x-4)}-\frac{2}{x+4}+\frac{3}{2(x-4)}$

$= \frac{2(3x)}{2(x+4)(x-4)}-\frac{2\times2(x-4)}{2(x+4)(x-4)}+\frac{3(x+4)}{2(x-4)(x+4)}$

$= \frac{6x-4(x-4)+3(x+4)}{2(x+4)(x-4)}$

$= \frac{6x-4x+16+3x+12}{2(x+4)(x-4)}$

$= \frac{5x+28}{2(x+4)(x-4)}$

H) Solving Equations involving Algebraic Fractions

Solve the following equations

a) $\frac{3}{2p-1}-\frac{4}{5p+4}=0$

b) $\frac{6}{x+2}-1=\frac{x+1}{2}$

c) $\frac{x}{x^2-1}-\frac{4x}{(3x^2-x-2)}+\frac{1}{x+1}=0$

a) $\frac{3}{2p-1}-\frac{4}{5p+4}=0$

$\frac{3}{2p-1}=\frac{4}{5p+4}$

$3(5p+4)=4(2p-1)$ (Cross Multiply)

$15p+12=8p-4$

$7p=-16$

$p=-\frac{16}{7}$

b) $\frac{6}{x+2}-1=\frac{x+1}{2}$

$\frac{6-(x+2)}{x+2}=\frac{x+1}{2}$

$\frac{4-x}{x+2}=\frac{x+1}{2}$

$2(4-x)=(x+1)(x+2)$

$8-2x=x^2+3x+2$

$x^2+5x-6=0$

$(x-1)(x+6)=0$

$x=1$ or $x=-6$

c) $\frac{x}{x^2-1}-\frac{4x}{(3x^2-x-2)}+\frac{1}{x+1}=0$

$\frac{x}{(x+1)(x-1)}-\frac{4x}{(3x+2)(x-1)}+\frac{1}{x+1}=0$

$\frac{x(3x+2)-4x(x+1)+(x-1)(3x+2)}{(x+1)(x-1)(3x+2)}=0$

$\frac{3x^2+2x-4x^2-4x+3x^2-3x+2x-2}{(x+1)(x-1)(3x+2)}=0$

$\frac{2x^2-3x-2}{(x+1)(x-1)(3x+2)}=\frac{0}{1}$

$2x^2-3x-2=0$

$(2x+1)(x-2)=0$

$x=-\frac{1}{2}$ or $x=2$

Note that after cross multiply, the left side denominator becomes zero when multiplied with 0

APEXMATH TUITION

I) Dealing with Fraction over fractions

When dealing with a multi-layered fractions, always turn the "longest line" into a divide sign.

$\frac{\frac{2}{3}}{x} = \frac{2}{3} \div x = \frac{2}{3} \times \frac{1}{x} = \frac{2}{3x}$

$\frac{2}{\frac{3}{x}} = 2 \div \frac{3}{x} = 2 \times \frac{x}{3} = \frac{2x}{3}$

$\frac{\frac{2}{x}}{\frac{y}{3}} = \frac{2}{x} \div \frac{y}{3} = \frac{2}{x} \times \frac{3}{y} = \frac{6}{xy}$

J) Handling Fraction over fractions

Simplify the following expressions:

a) $\frac{\frac{1}{x}-\frac{1}{x^3}}{\frac{1}{x^2}-1}$ **b)** $\frac{\frac{1}{q}+p}{\frac{1}{2p}+\frac{3}{q}}$

a) $\frac{\frac{1}{x}-\frac{1}{x^3}}{\frac{1}{x^2}-1}$

$= \frac{\frac{x^2}{x^3}-\frac{1}{x^3}}{\frac{1}{x^2}-\frac{x^2}{x^2}}$ (Make same denominators)

$= \frac{\frac{x^2-1}{x^3}}{\frac{1-x^2}{x^2}}$

$= \frac{x^2-1}{x^3} \div \frac{1-x^2}{x^2}$ (Change longest line into ÷)

$= \frac{(x+1)(x-1)}{x^3} \times \frac{x^2}{(1-x)(1+x)}$

$= \frac{\cancel{(x+1)(x-1)}}{x^3} \times \frac{x^2}{-\cancel{(x-1)(1+x)}}$

$= -\frac{1}{x}$

b) $\frac{\frac{1}{q}+p}{\frac{1}{2p}+\frac{3}{q}}$

$= \frac{\frac{1}{q}+\frac{pq}{q}}{\frac{q}{2pq}+\frac{6p}{2pq}}$

$= \frac{\frac{1+pq}{q}}{\frac{q+6p}{2pq}}$

$= \frac{1+pq}{q} \div \frac{q+6p}{2pq}$

$= \frac{1+pq}{\cancel{q}} \times \frac{2p\cancel{q}}{q+6p}$

$= \frac{2p(1+pq)}{q+6p}$

K) Changing the subject of a formula

Making a variable the subject of a formula means using algebraic manipulations to make the particular variable appear alone on the left side of the formula and no where else.

Refer to examples in the next box.

L) Changing the Subject of a Formula

Make the variable in the brackets the subject of the formula in each of the following cases.

a) $p = \frac{3-2h}{h+5}$ $[h]$

b) $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ $[y]$

c) $w = \frac{1}{2}h(a^2+c)$ $[a]$

d) $x = \sqrt{\frac{y+3z}{2y-z}}$ $[z]$

a) $p = \frac{3-2h}{h+5}$

$ph + 5p = 3 - 2h$ (cross multiply)

$ph + 2h = 3 - 5p$ (Terms with h to the left. The rest to right)

$h(p+2) = 3 - 5p$ (factorize out h)

$h = \frac{3-5p}{p+2}$ (divide $p+2$ on both sides)

b) $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$

$\frac{y-x}{xy} = \frac{1}{z}$ (Combine into single fraction)

$z(y-x) = xy$ (cross multiply)

$zy - zx = xy$ (expand)

$zy - xy = zx$ (terms with y to the left)

$y(z-x) = zx$ (factorize y)

$y = \frac{zx}{z-x}$ (divide $z-x$ on both sides)

c) $w = \frac{1}{2}h(a^2+c)$

$2w = ha^2 + hc$

$2w - hc = ha^2$

$a^2 = \frac{2w-hc}{h}$

$a = \pm\sqrt{\frac{2w-hc}{h}}$ (Rmb to put $\pm$ when square root both sides)

d) $x = \sqrt{\frac{y+3z}{2y-z}}$ (Square both sides)

$x^2 = \frac{y+3z}{2y-z}$

$x^2(2y-z) = y + 3z$

$2yx^2 - zx^2 = y + 3z$

$2yx^2 - y = 3z + zx^2$

$2yx^2 - y = z(3+x^2)$

$z = \frac{2yx^2-y}{3+x^2}$

M) Common Error

From $2a + b = \sqrt{3+z}$, make z the subject.

$2a + b = \sqrt{3+z}$

$4a^2 + b^2 = 3 + z$

$z = 4a^2 + b^2 - 3$

When squaring both sides, you square the "whole left" and "whole right" together. We are not allowed to square individual terms!

$2a + b = \sqrt{3+z}$

$(2a+b)^2 = 3 + z$

$z = (2a+b)^2 - 3$

Do recall that $(2a+b)^2$ is NOT equals to $4a^2 + b^2$

N) Changing the Subject of a Formula (*Advanced)

Express z as the subject of the formula for each case

a) $y = 2z + \sqrt{4x + yz^2 - 4yz}$

b) $A = 3pr\sqrt{z^2 - r^2}$

Always move the terms that are not under the square root to one side first. Then squaring both sides.

a) $y = 2z + \sqrt{4x + yz^2 - 4yz}$

$y - 2z = \sqrt{4x + yz^2 - 4yz}$

$(y-2z)^2 = 4x + yz^2 - 4yz$

$y^2 + 4z^2 - 4yz = 4x + yz^2 - 4yz$

$4z^2 - 4yz - yz^2 + 4yz = 4x - y^2$

$4z^2 - yz^2 = 4x - y^2$

$z^2(4-y) = 4x - y^2$

$z^2 = \frac{4x-y^2}{4-y}$

$z = \pm\sqrt{\frac{4x-y^2}{4-y}}$

b) $A = 3pr\sqrt{z^2 - r^2}$

$\frac{A}{3pr} = \sqrt{z^2 - r^2}$

$\frac{A^2}{9p^2r^2} = z^2 - r^2$

$z^2 = \frac{A^2}{9p^2r^2} + r^2$

$z = \pm\sqrt{\frac{A^2}{9p^2r^2} + r^2}$

O) Common Error

From $\sqrt{a^2 - b^2} = c$, make a the subject.

$\sqrt{a^2 - b^2} = c$

$a - b = c$

$a = c + b$

Note that $\sqrt{a^2 - b^2} \neq a - b$

$\sqrt{a^2 - b^2} = c$

$a^2 - b^2 = c^2$

$a^2 = c^2 + b^2$

$a = \pm\sqrt{c^2 + b^2}$

*Rmb to put $\pm$ whenever square rooting both sides of equation.

P) Changing the Subject of a Formula (*Advanced)

i) Given that $2ax + b = \sqrt{b^2 - 4ac}$, make c the subject of the formula and show that your answer can be factorised to $c = -x(ax+b)$

ii) Hence, find the value of c when $x = 1, a = 2$ and $b = -5$

i) $2ax + b = \sqrt{b^2 - 4ac}$,

$(2ax+b)^2 = b^2 - 4ac$

$4ac = b^2 - (2ax+b)^2$

$c = \frac{b^2-(2ax+b)^2}{4a}$

$c = \frac{(b+(2ax+b))(b-(2ax+b))}{4a}$

$c = \frac{(b+2ax+b)(b-2ax-b)}{4a}$

$c = \frac{(2ax+2b)(-2ax)}{4a}$

$c = \frac{\cancel{2}(ax+b)(-\cancel{2a})(x)}{\cancel{4a}}$

$c = -x(ax+b)$

ii) $c = -x(ax+b)$

$c = -(1)[(2)(1) + (-5)]$

$c = 3$

Single Fraction without "Plus and Minus"

Simplify $\frac{(2a^3b)^2}{8ab^5}$

Algebraic Fractions (Single Fraction)

Simplify each of the following.

a) $\frac{3x}{6x^2+5x}$

b) $\frac{6m^2n-9mn^2}{2m^2-3mn}$

c) $\frac{18x^2-50}{6x^2-x-15}$

d) $\frac{2a^2-ac-c^2}{a^2+ab-ac-bc}$

*e) $\frac{(3x+2y)^2-6x-4y}{9x^2-4y^2}$ (Advanced)

Multiplication and Division of Algebraic Fractions

Simplify each of the following.

a) $\frac{5a^2b^2}{18c} \times \frac{2b^2c^3}{15a(2ab^2)}$

b) $\frac{y^2-y-2}{2-6y} \div \frac{3y^2-12}{2y+4} \times \frac{1}{y+1}$

c) $\frac{2y^2-3y-5}{y^2-1} \times \frac{(y-1)^2}{y-1}$

*d) $\left(n-\frac{9}{n}\right) \div \left(11-2n-\frac{15}{n}\right)$

(Advanced)

Addition and Subtraction of Algebraic Fractions

Simplify each of the following.

a) $2-\frac{3n+4}{12n}-\frac{5}{4n}$

b) $\frac{4}{x-3}+\frac{6}{x+3}$

c) $\frac{-18y}{y^2-3y-18}-\frac{2y}{y+3}$

d) $\frac{3x}{x^2-16}-\frac{2}{x+4}-\frac{3}{8-2x}$

Solving Equations involving Algebraic Fractions

Solve the following equations

a) $\frac{3}{2p-1}-\frac{4}{5p+4}=0$

b) $\frac{6}{x+2}-1=\frac{x+1}{2}$

c) $\frac{x}{x^2-1}-\frac{4x}{(3x^2-x-2)}+\frac{1}{x+1}=0$

Handling Fractions over fractions

Simplify the following expressions:

a) $\frac{\frac{1}{x}-\frac{1}{x^3}}{\frac{1}{x^2}-1}$

b) $\frac{\frac{1}{q}+p}{\frac{1}{2p}+\frac{3}{q}}$

Changing the Subject of a Formula

Make the variable in the brackets the subject of the formula in each of the following cases.

a) $p=\frac{3-2h}{h+5}$ $[h]$

b) $\frac{1}{x}-\frac{1}{y}=\frac{1}{z}$ $[y]$

c) $w=\frac{1}{2}h(a^2+c)$ $[a]$

d) $x=\sqrt{\frac{y+3z}{2y-z}}$ $[z]$

Changing the Subject of a Formula (*Advanced)

Express z as the subject of the formula for each case

a) $y=2z+\sqrt{4x+yz^2-4yz}$

b) $A=3pr\sqrt{z^2-r^2}$

Changing the Subject of a Formula (*Advanced)

i) Given that $2ax+b=\sqrt{b^2-4ac}$, make c the subject of the formula and show that your answer can be factorised to $c=-x(ax+b)$

ii) Hence, find the value of c when $x=1, a=2$ and $b=-5$