Sec 2 Math: Direct Inverse Proportion

A) Direct Proportion

Description	Equation Form	Ratio Form
y is directly proportional to x	$y = kx$	$\frac{y_2}{y_1} = \frac{x_2}{x_1}$
y is directly proportional to x^2	$y = kx^2$	$\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^2$
y is directly proportional to cube root of x	$y = k\sqrt[3]{x}$	$\frac{y_2}{y_1} = \sqrt[3]{\frac{x_2}{x_1}}$

B) Inversely Proportion

Description	Equation Form	Ratio Form
y is inversely proportional to x	$y = \frac{k}{x}$	$\frac{y_2}{y_1} = \frac{x_1}{x_2}$
y is inversely proportional to $\sqrt{x}$	$y = \frac{k}{\sqrt{x}}$	$\frac{y_2}{y_1} = \sqrt{\frac{x_1}{x_2}}$
y is inversely proportional to square of x	$y = \frac{k}{x^2}$	$\frac{y_2}{y_1} = \left(\frac{x_1}{x_2}\right)^2$

C) Direct Proportion example (Basic)

Given that y is directly proportional to $\sqrt{x}$ and $y = 10$ when $x = 25$. Find
a) an equation connecting x and y,
b) the value of y when $x = \frac{1}{4}$,
c) the value of x when $y = 18$.

a) $y = k\sqrt{x}$
Sub $y = 10, x = 25$:
$10 = k\sqrt{25}$
$10 = k(5)$
$k = \frac{10}{5} = 2$
$\therefore y = 2\sqrt{x}$
b) Sub $x = \frac{1}{4}$:
$y = 2\sqrt{\frac{1}{4}}$
$y = 2\left(\frac{1}{2}\right) = 1$
c) Sub $y = 18$:
$18 = 2\sqrt{x}$
$\sqrt{x} = 9$
$x = 9^2 = 81$

D) Inverse Proportion example (Basic)

Given that y varies inversely as the cube of x and $y = 7$ when $x = 2$, find
a) an equation connecting x and y,
b) the value of y when $x = \frac{1}{3}$,
c) the value of x when $y = 7000$.

a) $y = \frac{k}{x^3}$
Sub $y = 7, x = 2$:
$7 = \frac{k}{2^3}$
$k = 7 \times 2^3 = 56$
$\therefore y = \frac{56}{x^3}$
b) Sub $x = \frac{1}{3}$:
$y = \frac{56}{\left(\frac{1}{3}\right)^3}$
$y = 1512$
c) Sub $y = 7000$:
$7000 = \frac{56}{x^3}$
$7000x^3 = 56$
$x^3 = \frac{56}{7000}$
$x = \sqrt[3]{\frac{56}{7000}}$
$x = \frac{1}{5}$

E) Man-hours Concept

- One man-hour is defined as the amount of work that is done by 1 man in 1 hour
- no. of men $\times$ no. of hours worked = no. of man-hours
- man-hour concept can be used for other scenarios such as "man-days", "worker-hours", "tap-hours" etc.

F) Man-hours Example (Intermediate)

Twelve workers can build a wall in 9 days. Assuming that they all work at the same rate, find
i) the number of days needed by ten workers to build the wall,
ii) the number of workers needed if the wall is to build in 6 days.
iii) the number of workers needed to build 3 walls in 12 days.
Mr tan hired twelve workers to build one wall. After 3 days, two workers left.
iv) How many days would the remaining ten workers take to finish building the wall?

i) Amount of work required for 1 wall $= 12 \times 9 = 108$ men-days
Number of days needed by 10 workers $= \frac{108\ men-days}{10\ men} = 10.8\ days$
ii) Number of workers needed $= \frac{108\ men-days}{6\ days} = 18\ men$
18 workers required.
iii) 1 Wall require 108 men-days,
3 walls require $108 \times 3 = 324$ men-days
Number of workers needed $= \frac{324\ men-days}{12\ days} = 27\ men$
27 workers is needed.
iv) Amount of work required $= 108$ men-days
Amount of work done in first 3 days $= 12 \times 3 = 36$ men-days
Amount of work left $= 108 - 36 = 72$ men-days
Number of days required $= \frac{72\ men-days}{10\ men} = 7.2\ days$

G) Three Variable Example (Intermediate)

12 workers together completed sewing 30 teddy bears in 6 days.
i) How many workers would be required to complete sewing 15 teddy bears in 4 days?
ii) What is the assumption made?

i) **Identify that Workers (W) are directly proportional to teddy bears (B) made but inversely proportional to Days (D) required.

$\therefore W = \frac{kB}{D}$ *We can chain variables together (direct on top, inverse below)
$12 = \frac{k(30)}{6}$
$k = 12 \times \frac{6}{30} = 2.4$
$\therefore W = \frac{2.4B}{D}$
Sub $B = 15, D = 4$: $W = \frac{2.4(15)}{4} = 9$
9 workers are required.

ii) The assumption is that all the workers work at the same rate.

H) Prove/Show proportion

If y is directly proportional to x, it means $y = kx$ or $\frac{y}{x} = k$
so, $\frac{y}{x}$ **is a constant.**
If y is inversely proportional to x, it means $y = \frac{k}{x}$ or $yx = k$
so, yx **is a constant.**
If y is directly proportional to square of x, it means $y = kx^2$ or $\frac{y}{x^2} = k$
so, $\frac{y}{x^2}$ **is a constant.**

I) Prove/Show proportion

The table show that Area (A) and the Length (L) of various figures

Area (A)	200	50	12.5	8
Length (L)	1	2	4	5

i) Is A directly proportional to L? Show your working clearly
ii) Is A inversely proportional to L? Show your working clearly
iii) Is A inversely proportional to L^2? Show your working clealy

i) If A is directly proportional to L, $\frac{A}{L} = constant.$

$\frac{A}{L}$	$\frac{200}{1} = 20$	$\frac{50}{2} = 25$	$\frac{12.5}{4} = 3.125$	$\frac{8}{5} = 1.6$

Since $\frac{A}{L}$ is not a constant, A is **not directly proportional** to L.

ii) If A is inversely proportional to L, $AL = constant$.

$A \times L$	$200 \times 1 = 200$	$50 \times 2 = 100$	$12.5 \times 4 = 50$	$8 \times 5 = 40$

Since $A \times L$ is not a constant, A is **not Inversely proportional** to L.

iii) If A is inversely proportional to L^2, $AL^2 = constant$

$A \times L^2$	$200 \times 1^2 = 200$	$50 \times 2^2 = 200$	$12.5 \times 4^2 = 200$	$8 \times 5^2 = 200$

Since $A \times L^2$ is a constant, A is **Inversely proportional** to L^2.

J) Given difference of y for two different x values Example (Advanced)

It is given that y is inversely proportional to the square root of x. The difference between the values of y when $x = 4$ and $x = 16$ is 3.
i) Form an equation in terms of x and y
ii) Hence, find the value of y when $x = 256$.

i) $y = \frac{k}{\sqrt{x}}$
$y_1 = ?$ $x_1 = 4$
$y_2 = ?$ $x_2 = 16$
$y_1 = \frac{k}{\sqrt{4}}$
$y_2 = \frac{k}{\sqrt{16}}$
$y_1 - y_2 = 3$
$\frac{k}{\sqrt{4}} - \frac{k}{\sqrt{16}} = 3$
$\frac{k}{2} - \frac{k}{4} = 3$ (larger – smaller)
$\frac{2k-k}{4} = 3$
$k = 12$
$\therefore y = \frac{12}{\sqrt{x}}$
ii) Sub $b = 256$:
$y = \frac{12}{\sqrt{256}} = \frac{3}{4}$

K) Direct Proportion Given only one value Example (Intermediate)

A is directly proportional to the cube of M. It is given that $A = 3$ for a certain value of M. Find the value of A when this value of M increases by 150%.

Method 1 (Ratio form)

$\frac{A_2}{A_1} = \left(\frac{M_2}{M_1}\right)^3$

$A_1 = 3$ and $\frac{M_2}{M_1} = \frac{2.5\ unit}{1\ unit}$

$\frac{A_2}{3} = \left(\frac{2.5}{1}\right)^3$ (substitute accordingly)

$\frac{A_2}{3} = 15.625$

$A_2 = 46.875$

Method 2 (Equation Form)

$A = KM^3 = 3$ (**E**quation)

$A_{New} = K(2.5M)^3$ (**S**ubstitute)

$= K(15.625)M^3$ (**E**xpand)

$= 15.625\ KM^3$ (**S**huffle)

$= 15.625\ (3)$ (**S**ubstitute)

$= 46.875$

****Note: Remember the steps "E S E S S"**
(Prounounced as "Yes, Yessss")
Equation, **S**ubstitute, **E**xpand, **S**huffle, **S**ubstitute

*****Note:** For the first substitutition step, Remember to follow the rule "**PONI**" which stands for "**Power Outside, Number Inside**". This is very important to follow as most errors made are due to students putting the power or the number at the wrong place.*

****Note:** Increase by 150% means 2.5X because initial 100% Plus 150% is 250%.*

L) Inverse Proportion Given only one value Example (Intermediate)

It is given that A is inversely proportional to B^2. It is known that $A = 3$ for a particular value of B. Find the value of A when this value of B is halved.

Method 1 (Ratio form)

$\frac{A_2}{A_1} = \left(\frac{B_1}{B_2}\right)^2$ (*Note: Inverse proportion so $\frac{B_2}{B_1}$)

$A_1 = 3$ and $\frac{B_1}{B_2} = \frac{1\ unit}{0.5\ unit}$

$\frac{A_2}{3} = \left(\frac{1}{0.5}\right)^2$

$\frac{A_2}{3} = 4$

$A_2 = 12$

Method 2 (Equation Form)

$A = \frac{k}{B^2} = 3$ (**E**quation)

$A_{New} = \frac{k}{(0.5B)^2}$ (**S**ubstitute)

$= \frac{k}{0.25B^2}$ (**E**xpand)

$= \frac{1}{0.25}\left(\frac{k}{B^2}\right)$ (**S**huffle)

$= \frac{1}{0.25}\ (3)$ (**S**ubstitute)

$= 12$

M) Proportion Question Given no values example (Advanced)

It is given that y is inversely proportional to square root of x. When x is reduced by 64%, calculate the percentage change in y

Method 1 (Ratio Form)

$\frac{y_2}{y_1} = \sqrt{\frac{x_1}{x_2}}$

$\frac{y_2}{y_1} = \sqrt{\frac{1\ unit}{0.36\ unit}}$

$\frac{y_2}{y_1} = \frac{5}{3}$

$y_2 = \frac{5}{3}y_1$

$y_2 = 1.667y_1$

(*Observe that final y is 1.667 times of original y)

% change $= \big((1.667 - 1) \times 100\big)\%$

$= 66.7\%$

Method 2 (Equation Form)

$y = \frac{k}{\sqrt{x}}$

$y_{New} = \frac{k}{\sqrt{0.36x}}$

$= \frac{k}{0.6\sqrt{x}}$

$= \frac{1}{0.6}\left(\frac{k}{\sqrt{x}}\right)$

$= \frac{1}{0.6}y$

$= 1.6667y$

$\%\ change\ in\ y = \frac{y_{New} - y}{y} \times 100$

$= \frac{\frac{1}{0.6}y - y}{y} \times 100$

$= \frac{\frac{1}{0.6} - 1}{1} \times 100$

$= 66.7\%$

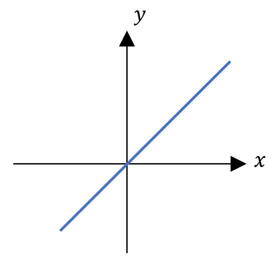
y increased by 66.7%.

***Note: % change =$\left(\frac{final - initial}{initial} \times 100\right)\%$**

N) Direct Proportion Graphs

Given y is directly proportional to x. Sketch the graph of y against x.

$y = kx$



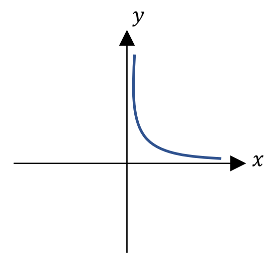
The $y = kx$ graph is a straight line.

****Note: the direct proportion graph ALWAYS passes through the origin!**

O) Inverse Proportion Graphs

Given y is inversely proportional to x. Sketch the graph of y against x.

$y = \frac{k}{x}$



Inverse proportion graph looks like a "L" shape.

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c) the value of x when $y = 18$

2) Given that y varies inversely as the cube of x and $y = 7$ when $x = 2$, find
a) an equation connecting x and y,
b) the value of y when $x = \frac{1}{3}$,
c) the value of x when $y = 7000$.

3) A is directly proportional to the cube of M. It is given that $A = 3$ for a certain value of M. Find the value of A when this value of M increases by 150%.

4) It is given that A is inversely proportional to B^2. It is known that $A = 3$ for a particular value of B. Find the value of A when this value of B is halved.

5) Twelve workers can build a wall in 9 days. Assuming that they all work at the same rate, find
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6) 12 workers together completed sewing 30 teddy bears in 6 days.
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7) It is given that y is inversely proportional to square root of x. When x is reduced by 64%, calculate the percentage change in y

8) It is given that y is inversely proportional to the square root of x. The difference between the values of y when $x = 4$ and $x = 16$ is 3.
i) Form an equation in terms of x and y
ii) Hence, find the value of y when $x = 256$.

9) Sketch the graph of $y = kx$ and the graph of $y = \frac{k}{x}$