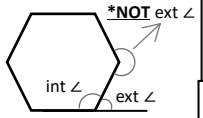
Sec 1 Math: Polygons

A) Polygon Terminologies and Formulae

Polygon: Closed figure with three or more sides
Regular Polygon: Polygon with **all sides** and **angles equal**

Interior Angle: refer to diagram (int ∠)
Exterior Angle: refer to diagram (ext ∠)
**Note: Please do not confuse exterior angle with reflex angle.*



Sum of exterior angles of any polygon = 360^o
Sum of interior angles n-sided polygon = $(n-2)\times 180^o$
**Note: It is often much easier to solve a question using sum of exterior angle property than interior angle formula*

B) Common Polygons and their Names

No. of Sides (n)	Name of Polygon	How to remember?
3	Triangle	You already know…
4	Quadrilateral	Quadbike has 4 wheels
5	Pentagon	Visualise holding a PEN with 5 fingers
6	Hexagon	Six and Hex both have X
7	Heptagon	Elder brother of Hexagon but without an X
8	Octagon	Octopus has eight legs
9	Nonagon	Nonagon and Nine both start with N
10	Decagon	A Decade is 10 years.

C) Interior angle Example (Basic)

a) Four of the interior angles of a pentagon are 105^o each. Calculate the fifth.
b) The exterior angles of a regular polygon is 40^o. What is the name of the polygon.

a) Sum of int angles of pentagon $= (5-2)\times 180^o$
$= 540^o$
Fifth angle $= 540^o - (4\times 105^o) = 120^o$

b) No. of sides $= \frac{360^o}{40^o} = 9$
It is a nonagon.

D) Regular Polygon Example (Intermediate)

Explain why the interior angle of a regular polygon cannot be 100^o

Exterior angle $= 180^o - 100^o = 80^o$
Number of sides $= \frac{360}{80} = 4.5$
4.5 is not an integer and a Polygon cannot have 4.5 sides

E) Interior Angles Examples (Intermediate)

Five interior angles in a heptagon are 130^o each. The remaining 2 angles are in the ratio of $1:4$. Find the largest exterior angle of this heptagon.

Sum of Interior angles $= (7-2)\times 180^o = 900^o$
Remaining angles $= 900 - (5\times 130^o) = 250^o$
Smaller interior angle $= \frac{250^o}{5} = 50^o$
Larger interior angle $= 50\times 4 = 200^o$
Largest exterior angle $= 180^o - 50^o$
$= 130^o$

**Note: Always read the question carefully (esp the word "interior" and "exterior"). Many students make the mistake of assuming question asking for largest interior angle*

F) Incomplete Figure Example (Intermediate)

The diagram shows a figure made up of a square, regular hexagon and an incomplete n-sided regular polygon. Find the value of n.

Each int angle in square $= 90^o$
Each int angle in hexagon $= \frac{(6-2)\times 180}{6} = 120^o$
Each int angle in n-sided polygon
$= 360^o - 90^o - 120^o$ (Angles at a point)
$= 150^o$
Each ext angle $= 180^o - 150^o = 30^o$
$n = \frac{360^o}{30^o} = 12$

G) Fitting Tile Example (Intermediate)

Explain why regular hexagonal tiles can fit together on a floor without gaps but a regular pentagonal tiles cannot.

Interior angle of Hexagon $= \frac{(6-2)\times 180}{6} = 120^o$
Angle at a point is 360^o
$\frac{360^o}{120^o} = 3$
At an intersection, 3 regular hexagon tiles can be fitted together without gaps.

Interior angle of Pentagon $= \frac{(5-2)\times 180}{5} = 108^o$
$\frac{360^o}{108^o} = 3.33$
$360^o - 3\times 108^o = 36^o$
If three pentagon are fitted at an intersection, there will be a gap with an angle of 36^o.

H) Relating Exterior and Interior Angles Examples (Intermediate)

Each interior angle of an n-sided regular polygon is $\frac{3}{2}$ of the size of each exterior angle. Find n.

**Key Concept: Interior + exterior angles $= 180^o$*
Let $ext\angle$ be x and $int\angle$ be $\frac{3}{2}x$.
Since interior angle + exterior angle $= 180^o$
$x + \frac{3}{2}x = 180^o$
$\frac{5}{2}x = 180^o$
$x = 180^o \div \frac{5}{2} = 72^o$
Exterior angle $= 72$
Number of sides $= \frac{360^o}{72^o} = 5$

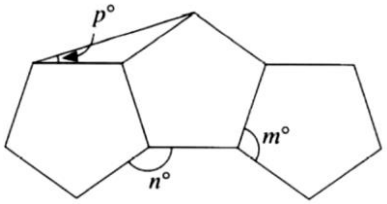
I) Interior Angles Examples (Intermediate)

A polygon has n sides. Three of its interior angles are 163^o, 137^o and 100^o. The remaining $(n-3)$ interior angles are 160^o each. Find n.

**Note: It is often more direct to calculate via exterior angles than interior angles*
$180^o - 163^o = 17^o$
$180^o - 137^o = 43^o$
$180^o - 100^o = 80^o$
Each remaining exterior angles are:
$180^o - 160^o = 20^o$
Since sum of exterior angles $= 360^O$,
$\frac{360^o - 17^o - 43^o - 80^o}{20^o} = 11$
$n = 11 + 3 = 14$

J) Forming Closed Ring Polygon (Intermediate)

a) The diagram shows three regular pentagons. Find the value of angles m, n and p.
b) Additional pentagons are added to the three pentagons to form a closed ring which is in the shape of a regular polygon. Find the number of additional pentagons needed.

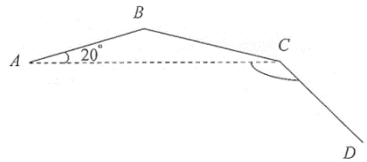


a) $m = \frac{(5-2)\times 180}{5} = 108^o$
$n = 360 - 108 - 108 = 144^o$
$p = \frac{180-144}{2} = 18^o$
**Note: Triangle at top is an isosceles triangle because the sides of the regular pentagons have same length.*

b) n^o is the interior angle of the closed ring polygon.
$\therefore$ Interior angle $= 144^o$
Exterior angle $= 180 - 144 = 36^o$
Number of angles is polygon $= \frac{360^o}{36^o} = 10$
Number of additional pentagons $= 10 - 3$
$= 7$

K) Part of Regular Polygon (Intermediate*)

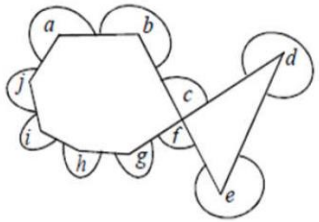
AB, BC and CD are adjacent sides of a regular polygon. Given that $\angle BAC = 20^o$, calculate
a) $\angle ABC$
b) the number of sides of the polygon



a) Since it is a regular polygon, $AB = BC = CD$
$\therefore \Delta ABC$ is an isosceles triangle.
$\angle ABC = 180^o - 20^o - 20^o$ (Isos Δ)
$= 140^O$
b) Exterior angle $= 180^O - 140^O = 40^O$
Number of sides of polygon $= \frac{360^O}{40^o} = 9$

L) Sum of Reflex angles (Advanced*)

The diagram shows an irregular heptagon and a triangle. Find the sum of all the marked angles.



Sum of Int angles in heptagon
$= (7 - 2) \times 180 = 900^o$

Sum of Int angles in triangle $= 180^o$

Since there are 9 corners,
Total sum of angles of all corners
$= 360^o \times 9 = 3240^o$

Sum of all marked angles $= 3240 - 900 - 180$
$\underline{= \mathbf{2160^o}}$

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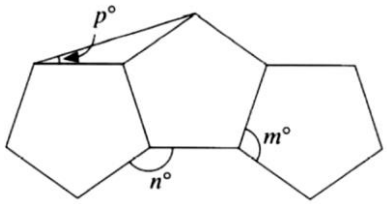

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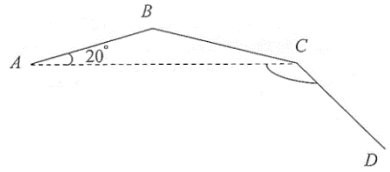

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