Sec 2 Math: Congruence and Similarity

A) Congruent Figures

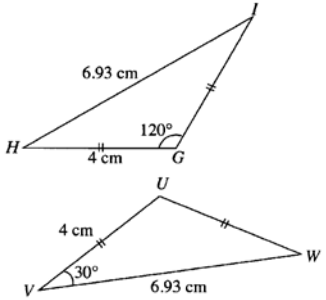
Congruent figures have **same shape** and **same size.**

To prove that 2 figures are congruent, we need to **prove** that:
1) **All corresponding angles are equal AND**
2) All corresponding sides are equal

$\Delta ABC \equiv \Delta XYZ$
The ' $\equiv$ ' sign means 'is congruent to'

B) Identifying Congruent Triangles and Writing Statement of Congruence

State and explain whether the following pair of triangles are congruent.

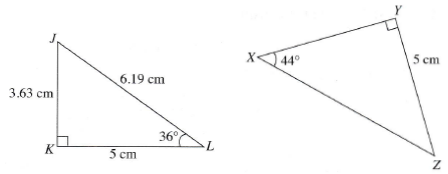


Note: Presentation is important!!

$\angle IHG = \angle HIG$ (base $\angle s$ of Isos Δ)
$= \frac{180^o - 120^o}{2}$
$= 30^o$
$\angle UVW = \angle UWV$ (base $\angle s$ of Isos Δ)
$= 30^o$
$\angle WUV = 180^o - 30^o - 30^o$ ($\angle$ sum of Δ)
$= 120^o$
$\angle IHG = \angle WVU = 30^o$
$\angle HIG = \angle VWU = 30^o$
$\angle HGI = \angle VUW = 120^o$
$GH = UV = 4\ cm$
$HI = VW = 6.93\ cm$
$GI = UW = 4\ cm$
Since all the corresponding sides and angles of ΔGHI and ΔUVW are the same, the two triangles have the same shape and size and are congruent.
*(*Upper Sec/IP please use ASA, SSS etc.)*

C) Writing Statement of Congruence

State and explain whether the following pair of triangles are congruent.

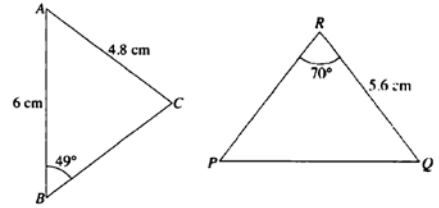


Note: Presentation is important!!

$\angle LJL = 180^o - 90^o - 36^o$ ($\angle$ sum of Δ)
$= 54^o$
$\angle XZY = 180^o - 90^o - 44^o$ ($\angle$ sum of Δ)
$= 46^o$
Since the angles in ΔJKL are not equal to the angles in ΔXYZ, the two triangles are not congruent.
*(*Upper Sec/IP please use RHS, SSS etc. instead)*

D) Problems involving Congruent Triangles

In the figures below, ΔABC ≡ ΔPQR.



Find i) length of BC ii) $\angle RPQ$

Since $\Delta ABC \equiv \Delta PQR$, the corresponding vertices match.

i) $BC = QR = 5.6\ cm$
ii) $\angle RQP = \angle CBA = 49^o$
$\angle RPQ = 180^o - 49^o - 70^o$ ($\angle$ sum of Δ)
$= 61^o$

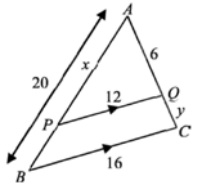
E) Similar Figures

Similar figures have **same shape** but not necessarily same size

To prove that 2 triangles are similar, we need to **prove** that either:
1) All corresponding angles are equal OR
2) All corresponding sides are in the same ratio

F) Problems involving Similar Figures

In the diagram, PQ is parallel to BC.



a) Identify the pair of similar triangles and explain why they are similar.
b) Hence, find the values of x and y.

Note: Presentation is important!!
a) ΔPAQ is similar to ΔBAC.
*(*It is wrong to say ΔPAQ is similar to ΔCBA. Do you know why?)*
$\angle PAQ = \angle BAC$ (Common $\angle$s)
$\angle APQ = \angle ABC$ (Corr $\angle$s)
$\angle AQP = \angle ACB$ (Corr $\angle$s)
Since all the corresponding angles are equal, ΔPAQ is similar to ΔBAC.

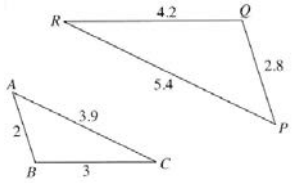
b) Since ΔPAQ is similar to ΔBAC, corresponding sides are in the same ratio:
$\frac{PA}{BA} = \frac{AQ}{AC} = \frac{PQ}{BC}$
$\frac{x}{20} = \frac{6}{6+y} = \frac{12}{16}$
$\frac{x}{20} = \frac{12}{16}$
$x = \frac{12}{16} \times 20$
$x = 15$

$\frac{6}{6+y} = \frac{12}{16}$
$6(16) = 12(6+y)$
$96 = 72 + 12y$
$12y = 24$
$y = 2$

$\therefore x = 15, y = 2$

G) Determine similarity

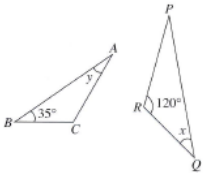
Determine whether ΔABC is similar to ΔPQR. Find x and y.



Note: Presentation is important!!
$\frac{AB}{PQ} = \frac{2}{2.8} = \frac{5}{7}$
$\frac{BC}{QR} = \frac{3}{4.2} = \frac{5}{7}$
$\frac{AC}{PR} = \frac{3.9}{5.4} = \frac{13}{18}$
Since $\frac{AB}{PQ} \neq \frac{AC}{PR}$, the corresponding sides are not in the same ratio. Hence, ΔABC is **not** similar to ΔPQR.

H) Angles in Similar Figures

ΔABC is similar to ΔPQR. Find x and y.



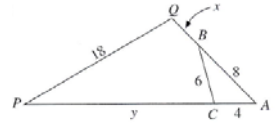
**Note that corresponding angles of similar triangles are EQUAL.*
$\angle PQR = \angle ABC = 35^o$
$x = 35^o$
$\angle BCA = \angle QRP = 120^o$
$y = 180^o - 35^o - 120^o$ ($\angle$ sum of Δ)
$y = 25^o$

I) Similar Figures Tips**

If question states that ΔABC is similar to ΔXYZ, we can straight away write $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$ without looking at the diagram. This is especially useful if the diagram is confusing or if complex algebra is involved. *Refer to next example

J) Problems on Similar Triangles

ΔABC is similar to ΔAPQ, find x and y.



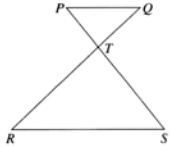
$\frac{AB}{AP} = \frac{BC}{PQ} = \frac{AC}{AQ}$ (*follow order given in Qn)
$\frac{8}{y+4} = \frac{6}{18} = \frac{4}{x+8}$ (*fill in base on diagram)

$\frac{8}{y+4} = \frac{6}{18}$
$8(18) = 6(y+4)$
$144 = 6y + 24$
$y = 20$

$\frac{6}{18} = \frac{4}{x+8}$
$6(x+8) = 4(18)$
$6x + 48 = 72$
$x = 4$

K) Problems on Similar Figures

ΔPQT is similar to ΔSRT. Given that $\frac{PT}{ST} = \frac{2}{3}$ and $RQ = 15$cm, calculate the length of QT.



Let QT be x.
$\frac{PQ}{SR} = \frac{QT}{RT} = \frac{PT}{ST}$ (*follow order given in Qn)
$\frac{PQ}{SR} = \frac{x}{15-x} = \frac{2}{3}$
$\frac{x}{15-x} = \frac{2}{3}$
$3x = 30 - 2x$
$5x = 30$
$x = 6$
QT is 6cm.

L) Similar Objects

The two vases are similar. The height of the smaller vase is 16cm and the larger vase is 20cm.

i) The base radius of the smaller vase is 2cm. Find the base radius of the larger vase.

ii) The base circumference of the larger vase is 100cm, find the base circumference of the smaller vase.

i) Scale factor of enlargement

$= \frac{Large}{small} = \frac{20}{16}$

$= \frac{5}{4}$

$\frac{Large\ Radius}{Small\ Radius} = \frac{5}{4}$

$\frac{Large\ Radius}{2} = \frac{5}{4}$

$Large\ Radius = \frac{5}{4} \times 2$

$Large\ Radius = 2.5\ cm$

ii) $\frac{Large\ Circumference}{Small\ Circumference} = \frac{5}{4}$

$\frac{100}{Small\ Circumference} = \frac{5}{4}$

$400 = 5 \times Small\ circumference$

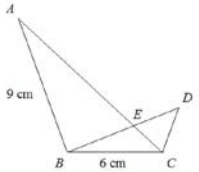
$Small\ Circumference = 80\ cm$

M) Scale Factor of Enlargement

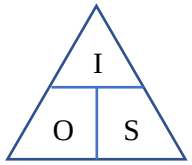
In the diagram, ΔABC is reduced to form ΔBCD.

a) State the scale factor of reduction.

b) Calculate the length of DC.



Hint: Remember using the triangle



I stands for Image,
O stands for Original and
S stands for Scale Factor.

Since question states that ΔABC is reduced to form ΔBCD, it means ΔABC is the orginal and ΔBCD is the image.

a) Scale factor $= \frac{Image\ length}{Object\ length} = \frac{BC}{AB} = \frac{6}{9} = \frac{2}{3}$

The scale factor of reduction is $\frac{2}{3}$.

b) $\frac{AB}{BC} = \frac{BC}{CD} = \frac{AC}{BD}$

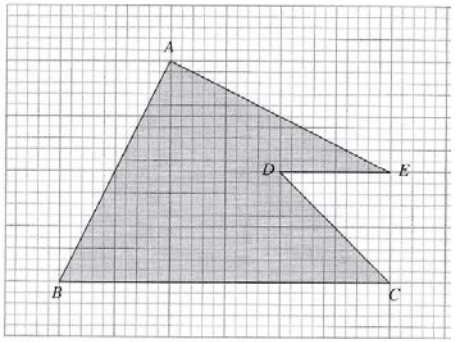
$\frac{9}{6} = \frac{6}{CD} = \frac{AC}{BD}$

$9(CD) = 6(6)$

$CD = 4$

N) Scale Drawings on Grid

Draw a reduction of figure $ABCDE$ using a scale factor of $\frac{1}{2}$.



Solution:

Step 1) Mark out a point A on the answer grid

Step 2) Count the movement from A to B in original diagram:
Movement from A to B is (2 units Left, 4 units down)

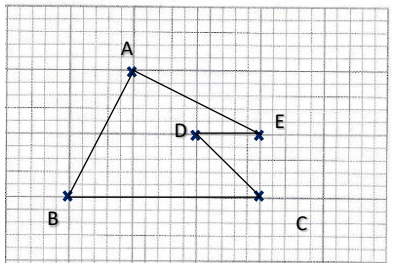
Step 3) Multiply movement by scale factor $\left(\frac{1}{2}\right)$.

Hence, new movement from A to B is (1 unit Left, 2 units down)

Step 4) Count new movement from A to locate new B. Mark down B.

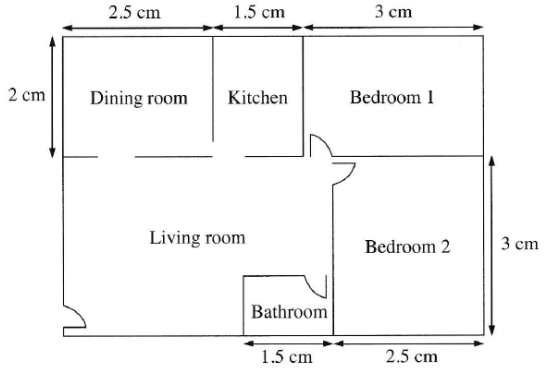
Step 5) Repeat Steps 2 to 4 to find points C, D and E.

Step 6) Connect all the points.



O) Scale Drawing / Floor Plan

The diagram below shows the floor plan of an apartment. The scale used is $1:150$.



a) Find the actual dimenions of Bedroom 1

b) Find the actual area, in square metres, of the Dining room

a) $1:150$

$1\ cm : 150\ cm$

$1\ cm : 1.5\ m$

Actual Length of Bedroom 1 $= 3 \times 1.5\ m$
$= 4.5\ m$

Actual Width of Bedroom 1 $= 2 \times 1.5\ m$
$= 3\ m$

∴ Actual dimensions of Bedroom 1 are $4.5\ m\ by\ 3\ m$.

b) **Method 1 (Preferred method)**

Actual Length of Dining room $= 2.5 \times 1.5\ m$
$= 3.75\ m$

Actual Width of Dining Room $= 2 \times 1.5\ m$
$= 3\ m$

Actual Area of Dining Room $= 3.75 \times 3$
$= 11.25\ m^2$

Method 2

Actual Length of Dining room $= 2.5 \times 150\ cm$
$= 375\ cm$

Actual Width of Dining Room $= 2 \times 150\ cm$
$= 300\ cm$

Actual Area of Dining Room $= 375 \times 300$
$= 112500\ cm^2$
$= 11.25\ m^2$

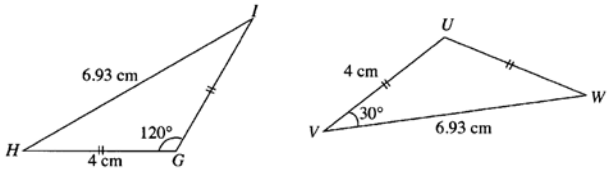
***Extra Notes (Common error):*

Although $1\ m = 100\ cm$,

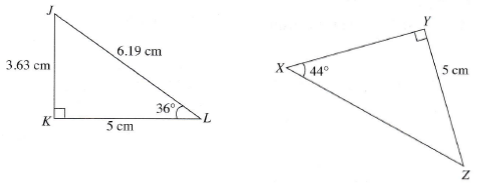
Do note that **$1\ m^2$** is **not equals** to **$100\ cm^2$**.

$1\ m^2$ is actually **equals** to **$100^2\ cm^2$** $(i.e.\ 10000\ cm^2)$.

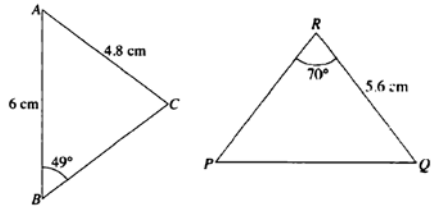
State and explain whether the following pair of triangles are congruent.



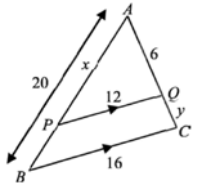
State and explain whether the following pair of triangles are congruent.



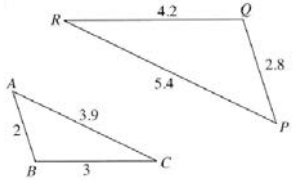
In the figures below, $\Delta ABC \equiv \Delta PQR$. **Find i) length of** BC **ii)** $\angle RPQ$



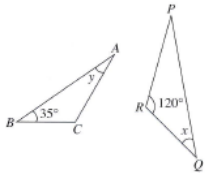
In the diagram below, PQ is parallel to BC.
a) Identify the pair of similar triangles and explain why they are similar.
b) Hence, find the values of x **and** y**.**



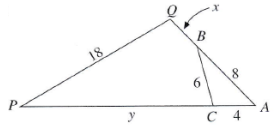
From the diagrams below, determine whether ΔPQR **is similar to** ΔABC**. Find** x **and** y**.**



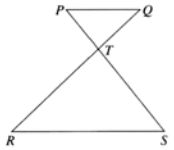
ΔABC **is similar to** ΔPQR**. Find** x **and** y**.**



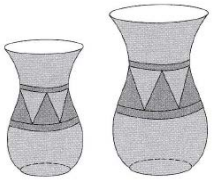
In the diagram below, ΔABC **is similar to** ΔAPQ**, find** x **and** y**.**



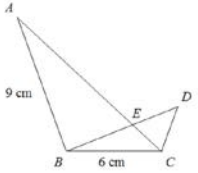
ΔPQT **is similar to** ΔSRT**. Given that** $\frac{PT}{ST} = \frac{2}{3}$ **and** $RQ = 15$**cm, calculate the length of** QT**.**



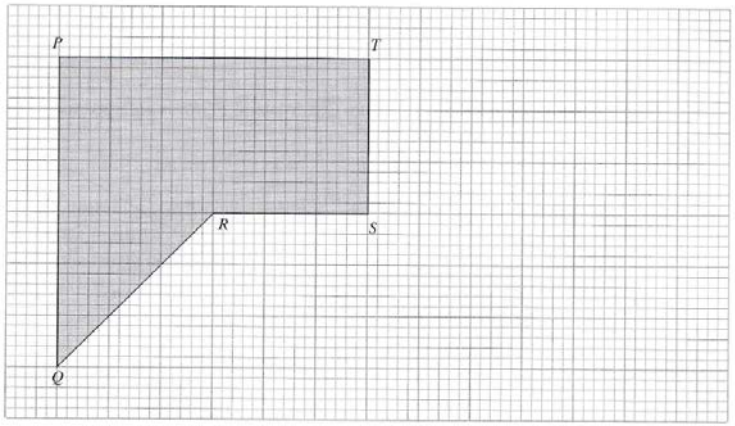
The two vases are similar. The height of the smaller vase is 16cm and the larger vase is 20cm.
i) The base radius of the smaller vase is 2cm. Find the base radius of the larger vase.
ii) The base circumference of the larger vase is 100cm, find the base circumference of the smaller vase.



In the diagram, ΔABC **is reduced to form** ΔBCD**.**
a) State the scale factor of reduction.
b) Calculate the length of DC**.**



Draw a reduction of figure $PQRST$ **using a scale factor of** $\frac{2}{3}$**.**



The diagram below shows the floor plan of an apartment. The scale used is $1:150$**.**
a) Find the actual dimenions of Bedroom 1
b) Find the actual area, in square metres, of the Dining room