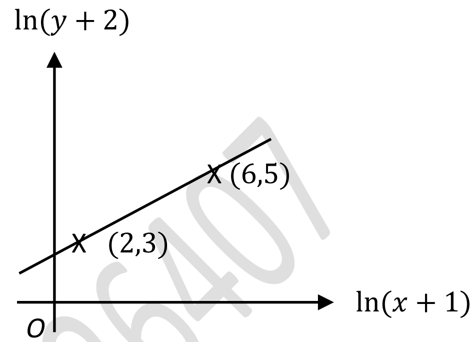
Linear Law

1 The diagram shows part of a straight line obtained when plotting values of $\ln(y+2)$ against $\ln(x+1)$. Express y in terms of x.



Gradient = $\frac{5-3}{6-2}=\frac{1}{2}$

Equation of line : $Y=\frac{1}{2}X+c$

Sub (2,3) into equation:

$3=\frac{1}{2}(2)+c$

$c=2$

Equation of line : $Y=\frac{1}{2}X+2$

$\ln(y+2)=\frac{1}{2}\times\ln(x+1)+2$

$\ln(y+2)=\ln(x+1)^{\frac{1}{2}}+\ln e^2$

$\ln(y+2)-\ln(x+1)^{\frac{1}{2}}=\ln e^2$

$\ln\frac{y+2}{\sqrt{x+1}}=\ln e^2$

$\frac{y+2}{\sqrt{x+1}}=e^2$

$y=e^2\sqrt{x+1}-2$

2 In each of the following, a and b are unknown constants. Express each of them into the form $Y=mX+c$, where X and Y are functions of x and/or y, and m and c are constants.

$y^b=10^{x+a}$

$$ya^x=b+2$$

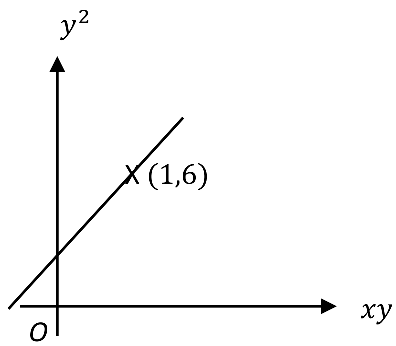
$$y=\frac{a}{\sqrt{x-b}}$$

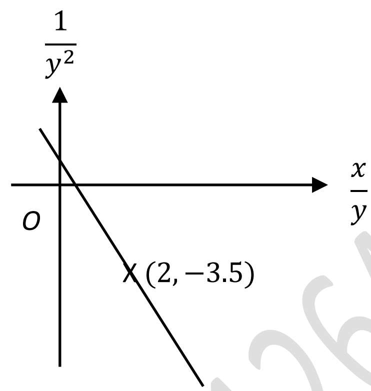
Ans: a) $\lg y=\frac{1}{b}x+\frac{a}{b}$

b) $\lg y=(-\lg a)x+\lg(b+2)$

c) $\frac{1}{y^2}=\frac{1}{a^2}x-\frac{b}{a^2}$

3 Alvin and Gina both used linear law to express the same equation into forms suitable for drawing straight line graphs. As they expressed the equation differently, 2 different graphs were obtained (as shown below). Determine the original equation relating x and y.





Solution:

$y^2 = m(xy) + c$ -(1)

Substitute Coordinates into Equation (1):

$(6) = m(1) + c$ -(2)

Divide equation (1) by y^2:

$1 = m\left(\frac{x}{y}\right) + c\left(\frac{1}{y^2}\right)$ -(3)

Substitute Coordinates into Equation (3):

$1 = m(2) + c(-3.5)$ -(4)

Simultaneously solve (2) and (4):

$m = 4$

$c = 2$

∴ Equation is $y^2 = 4(xy) + 2$