Sec 1 Math: Factors and Multiples

A) Prime Factorization

Prime factorize 600.

2	600
2	300
2	150
3	75
5	25
5	5
	1

$\therefore 600 = 2^3 \times 3 \times 5^2$

B) HCF/LCM (Basic Impt)

Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of 1764, 270 and 1800.

Prime factorize all 3 numbers:
$1764 = 2^2 \times 3^2 \times 7^2$
$270 = 2 \times 3^3 \times 5$
$1800 = 2^3 \times 3^2 \times 5^2$
(*HCF: Select only factors that appear in all numbers and take the smallest power*)
$HCF = 2 \times 3^2$
$= 18$

$1764 = 2^2 \times 3^2 \times 7^2$
$270 = 2 \times 3^3 \times 5$
$1800 = 2^3 \times 3^2 \times 5^2$
(*LCM: select all factors and select the largest power for each*)
$LCM = 2^3 \times 3^3 \times 5^2 \times 7^2$
$= 264600$

C) Rooting (Basic)

Use prime factorization, find
i) $\sqrt{676}$
ii) $\sqrt[3]{1728}$

i) $\sqrt{676} = \sqrt{2^2 \times 13^2} = 2 \times 13$
$= 26$

ii) $\sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3} = 2^2 \times 3$
$= 12$

D) Perfect Square/Cube (Intermediate)

a) Find the smallest value of n such that $2016n$ is a perfect cube.
b) Find the smallest value of k such that $\sqrt{2016k}$ is an integer.
c) Find the smallest value of m such that $\frac{2016}{m}$ is a square number.

$2016 = 2^5 \times 3^2 \times 7$
a) $2^5 \times 3^2 \times 7 \times n$ is a perfect cube.
$n = 2 \times 3 \times 7^2 = 294$
(n has to "provide" for each factor until their power is a multiple of 3)

b) $2^5 \times 3^2 \times 7 \times k$ is a perfect square.
$k = 2 \times 7 = 14$
(k has to "provide" for each factor until their power is a multiple of 2)

c) $\frac{2^5 \times 3^2 \times 7}{m}$ is a perfect square.
$m = 2 \times 7 = 14$
(m is used to "remove" from each factor until their power is a multiple of 2)

E) Multiple of another number (Intermediate)

Find the smallest positive integer value of n for which $198n$ is a multiple of 264.

Method 1
In other words, $198n$ is the lowest common multiple of 198 and 264.
Step 1) Find LCM of 198 and 264
$198 = 2 \times 3^2 \times 11$
$264 = 2^3 \times 3 \times 11$
$\text{LCM} = 2^3 \times 3^2 \times 11 = 792$
Step 2) Find n.
$198n = 792$
$n = \frac{792}{198} = 4$

Method 2
We want $\frac{198n}{264}$ to get an integer
$\frac{198n}{264} = \frac{2 \times 3^2 \times 11 \times n}{2^3 \times 3 \times 11}$
We need the powers of the top numbers to be at least equal or bigger than the bottom.
$\therefore n = 2^2$
$n = 4$

F) Word Problems (Intermediate)

The time taken for three boys to walk briskly around a field once are 1 min, 1min 15 sec and 1 min 30 sec respectively. If they start together at 3pm at the starting line, at what time will they next meet one another at the start line again?

This question is expecting you to find the Lowest Common Multiple. Do you know why?

Step 1) Prime factorize the three values.

1min	= 60 sec	$= 2^2 \times 3 \times 5$
1min 15 sec	= 75 sec	$= 3 \times 5^2$
1min 30 sec	= 90 sec	$= 2 \times 3^2 \times 5$

Step 2) Find LCM
LCM $= 2^2 \times 3^2 \times 5^2 = 900$

$900s = 15\ min$

3pm+15min = 3.15pm

The three boys will next meet at the starting line again at 3.15pm

G) Packing Word Problems (Intermediate)

Julie needs to pack 140 stalks of roses, 168 stalks of lilies and 210 stalks of orchids into identical baskets so that each type of flower is equally distributed among the baskets. Find
i) the largest number of baskets that can be packed
ii) the number of roses in the basket

This question is expecting you to find the Highest Common Factor. Do you know why?

Step 1) Prime factorize the three values
$140 = 2^2 \times 5 \times 7$
$168 = 2^3 \times 3 \times 7$
$210 = 2 \times 3 \times 5 \times 7$

Step 2) Find HCF
HCF $= 2 \times 7 = 14$

The largest number of baskets that can be packed is 14.

ii) $\frac{140}{14} = 10$
There are 10 roses in the basket.

H) Square Tile Word Problems (Intermediate)

A rectangular wall measuring 234 cm by 486 cm is to be completely covered by square tiles of the same size. Find
i) the greatest possible length of each square tile,
ii) the number of square tiles needed to cover the wall

This question is expecting you to find the Highest Common Factor. Do you know why?
Square tiles indicates that the length and breadth must be the same value.

Step 1) Prime factorize the two values.
$234 = 2 \times 3^2 \times 13$
$486 = 2 \times 3^5$

Step 2) Find HCF
HCF $= 2 \times 3^2 = 18$

i) The greatest possible length of each square tile is 18cm.

ii) $234 \div 18 = 13$
$486 \div 18 = 27$
$13 \times 27 = 351$
The wall needs 351 square tiles to cover.

I) Explanation Question (Intermediate)

Given that $p = 3^3 \times 5^6$, explain why p is a perfect cube.

The powers of all the prime factors of p are multiples of 3.

Note: "All the numbers/factors are perfect cube" is* ***NOT* *an acceptable answer.*

J) Both Perfect Square and Cube (Intermediate)

Given that $M = 3^2 \times 5^9$, find the smallest value k such that $M \times k$ is both a perfect cube and a perfect square. Leave your answer in index notation.

To be a perfect square, the powers must be multiples of 2
To be a perfect cube, the powers must be multiples of 3
To be both a perfect square and cube, the powers must be multiples of 6.

$3^2 \times 5^9 \times k$ must be both perfect square and cube,
$k = 3^4 \times 5^3$
(k has to "provide" for each factor until their power is a multiple of 6)
****Note: Remember that the question wants us to leave the answer in index notation. Please DO NOT evaluate the value itself.***

K) Perfect Cube (Intermediate)

p and q are prime numbers. Find the smallest values of p and q so that $360 \times \frac{p}{q}$ is a perfect cube.

Prime Factorize $360 = 2^3 \times 3^2 \times 5$

$\frac{360p}{q} = \frac{2^3 \times 3^2 \times 5 \times p}{q}$

p is multiplied and q is divided. This mean that p will "add in" new factors while q will "remove" factors.

Notice that 2^3 is already a perfect cube. Nothing needs to be done to it.

3^2 is one more 3 away from being a perfect cube. So it is easier to "add in" one more 3.

5 is two more 5s away from being a perfect cube. So it is easier to "remove" the 5.

Hence, $p = 3$ and $q = 5$

L) Reverse (Advanced)

The highest common factor and the lowest common multiple of three numbers are $2^2 \times 3$ and $2^3 \times 3^2 \times 5$ respectively. Given that two of the numbers are 36 and 180, find the smallest possible value of the third number.

$36 = 2^2 \times 3^2$

$180 = 2^2 \times 3^2 \times 5$

Let the third number be x.

x must include a minimum of $2^2 \times 3$ because that is the HCF.

x cannot include 3^2 if not the HCF will contain 3^2.

x must include 2^3 since the LCM has 2^3 but the other two numbers do not.

x does not need to include 5 since we are looking for the smallest possible value of x and since 180 already contributed a 5 to fulfil the LCM.

Hence, the smallest possible value of the third number is:

$2^3 \times 3 = 24$

Self Practice

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