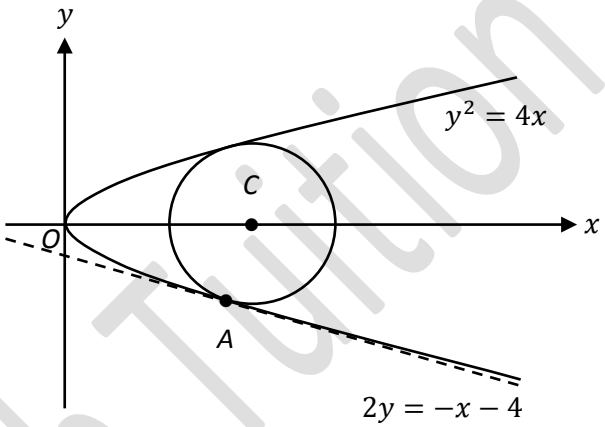
(8) Circles and Parabolas

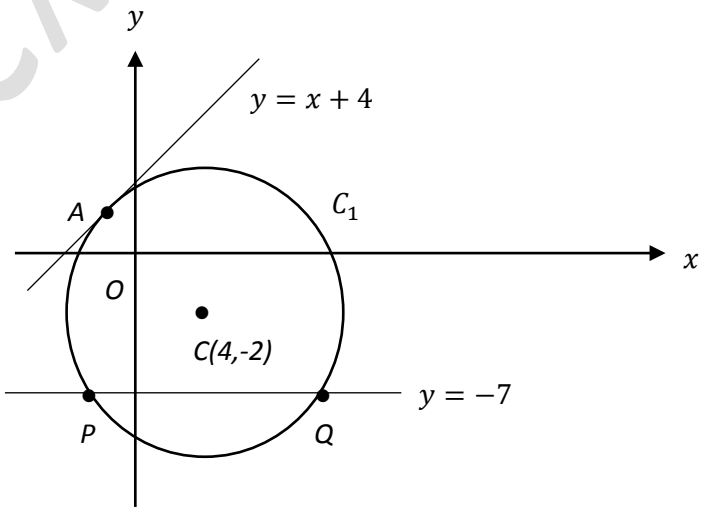
1. Find the possible equations of the circle which passes through the point (4,2) and touches both coordinate axes.

2. In the diagram (not drawn to scale), the circle with center C touches the curve $y^2 = 4x$ at the point A. $2y = -x - 4$ is the equation of the common tangent to both the circle and the curve at point A. Find
   i) the coordinates of point A
   ii) the equation of the circle

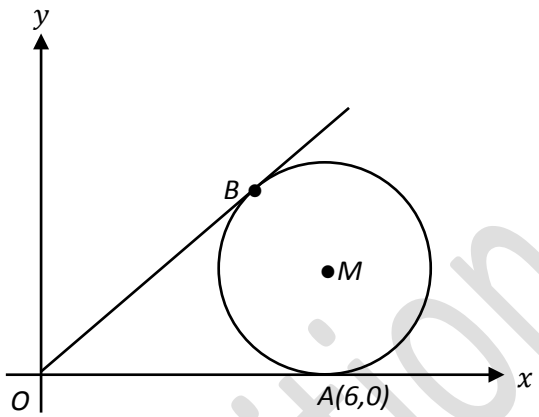


3. The diagram shows circle C_1 (not drawn to scale with the center of the circle at $C(4, -2)$. The line $y = x + 4$ is a tangent to circle at the point A.

   i) Find coordinates of A.
   ii) Find the equation of circle C_1.
   iii) Given that a line L: $y = -7$ intersects the circle C_1 at the points P and Q, find the coordinates of P and Q and find the shortest distance from the center of the circle to L.
   iv) Find the coordinates of the center of a second circle C_2 which has radius $\sqrt{89}$ units and also passes through P and Q. The center of the C_2 is lies above the x-axis.
   v) Find $\frac{area\ of\ C_1}{area\ of\ C_2}$

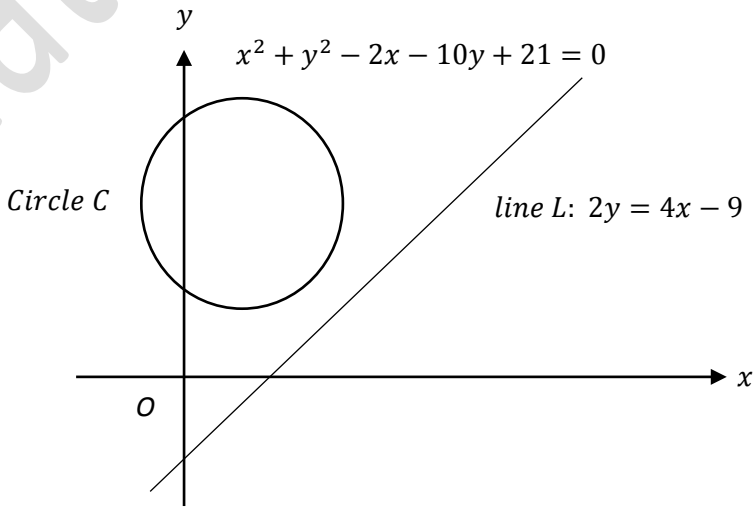


4. The diagram shows a circle C with center M and radius 2 units. The circle touches the x-axis at the point A(6,0) and the line OB is a tangent to the circle at the point B. Find
   i) the coordinates of center M,
   ii) the equation of the circle
   iii) the equation of the line OB.



5. Given that the circle passes through the points $A(1,6)$ and $B(5,8)$ and has radius 5, find the equation of the circle.

6. The diagram shows the circle C with equation $x^2 + y^2 - 2x - 10y + 21 = 0$ and the line L with equation $2y = 4x - 9$.
   i) Find the coordinates of the centre and the radius of the Circle C
   ii) Find the coordinates of the point on the line L which is closest to the Circle C
   iii) Find the equation of the circle which is a reflection of Circle C on the line L.



7. Find the equation of the circle that has its center at $(4,3)$ and has a tangent whose equation is given by $y = 3x + 1$

8. Find the equation of the circle which passes through the origin and the points $A(3,1)$ and $B(-1,-1)$.

9. Circle C has the equation $x^2 + y^2 + 6x - 6y - 7 = 0$.
   i) Find the coordinates of the center and the radius of the circle.
   ii) Point $A\ (-6, -1)$ is on the circle. The tangent of the circle at point A intersects the y −axis at Point B. Find the coordinates of Point B.
   iii) Show that circle C does not intersect the line $y = x - 3$.
   iv) Show that the origin lies inside circle C.
   v) Find the equation of the circle which is a reflection of circle C on the y −axis.

10. A, B and C are points with coordinates $(0, -1)$, $(3, -5)$ and $(10, -6)$ respectively. AD is a diameter of a circle with centre at B.
    i) Find the equation of the circle.
    ii) Show that CD is a tangent to the circle.
    iii) E is a point (4,2). Show that CE is another tangent to the circle and state the coordinates of the point of contact.