Sec 1 Math: Real Numbers

A) Real Numbers Definition

Prime Numbers: Positive integer that is divisible only by 1 and itself. (2,3,5,7,...)
The smallest prime number is 2!! (i.e. -3, 0 and 1 are NOT prime numbers)

Composite Numbers: Positive integer that have factors other than 1 and itself (4,6,8,9...)
Integers: Non-decimals (-2, -1, 0, 1, 2, 3...). ***Fun fact: "Integer" originated from latin "in + tangere" which means "intact" and hence refers to "whole" numbers.*
Whole Numbers: Positive integers including 0 (0, 1, 2, 3, ...)
Natural Numbers: All whole numbers but exclude 0. (1, 2, 3, 4, ...)
Perfect Square: An integer that is the square of another integer (1, 4, 9, 16, 25...)
**1 is considered a perfect square!*

Rational Number: A number that can be expressed as a fraction of two integers.
All integers, recurring numbers $(1.\dot{2}\dot{3}, 3.\dot{3}, \ldots)$ and terminating decimals (1.25, 1.62, -3.5 ...) are rational. $\frac{22}{7}$ is rational since it can be expressed as a fraction of 2 integers.
Irrational Number: A number that cannot be expressed as a fraction of two **integers**.
It is both non-terminating and non-recurring. Usually numbers that contain π or square roots of non-perfect squares ($\sqrt{2}, \sqrt{3}, \sqrt{8}, \ldots$) are irrational. ***Fun fact "Irrational" means impossible to be represented as a "ratio of two integers".*

B) Real Numbers Example

Consider the following set of numbers below:

-3 , $\sqrt{5}$, 9 , $\sqrt{25}$, $5.\dot{6}\dot{7}$, 0 , $\sqrt{0.04}$, 14 , π , $\frac{22}{7}$, 1 , 2

State all the numbers that are:

a) Integers **b) Irrational Numbers** **c) Composite numbers,**
d) Prime Numbers **e) Whole Numbers** **f) Rational Numbers**
g) Perfect Squares

a) Integers: $-3, 9, \sqrt{25}, 0, 14, 1, 2$
b) Irrational Numbers: $\sqrt{5}, \pi$
c) Composite Numbers: $9, 14$
d) Prime Numbers: $\sqrt{25}, 2$
e) Whole Numbers: $0, 1, 2, \sqrt{25}, 9, 14$
f) Rational Numbers: $-3, 9, \sqrt{25}, \ 5.\dot{6}\dot{7}\ , 0, \sqrt{0.04}, 14, \frac{22}{7}, 1, 2$
g) Perfect Squares: $9, 1$

C) Negative Numbers (Add/Subtract)

$7 + 12 = 19$ **Same sign** (both +), we **add** numbers.
$-7 - 12 = -19$ **Same sign** (both -), we **add** numbers.

$-12 + 7 = -5$ **Different sign** (+ and -), we **subtract** numbers.
$-7 + 12 = 5$ **Different sign** (+ and -), we **subtract** numbers.

D) Negative Numbers (Add/Subtract) Example

Find the value of each of the following:

a) $\mathbf{11 - 23}$ **b)** $\mathbf{-13 - 15}$ **c)** $\mathbf{8 + 7}$
d) $\mathbf{-8 + 11}$ **e)** $\mathbf{-4 - (-6)}$ **f)** $\mathbf{-6 - (-15)}$
a) -12 *b)* -28 *c)* 15 *d)* 3 *e)* 2 *f)* 9

E) BODMAS

Always do arithmetic following the order of BODMAS
Step 1: **(B)** Simplify expressions within **brackets** first.
Step 2: **(O)** Evaluate the **Order** (i.e power, square, square roots) next.
Step 3: **(DM)** Perform **Division and Multiplication** of numbers from left to right.
Step 4: **(AS)** Finally, perform **Addition and Subtraction** of numbers from left to right.

F) BODMAS Example

Without using a calculator, evaluate the value of
$(-2)^3 - 12 \div [2 - (\sqrt{25} + 3)]$

$(-2)^3 - 12 \div [2 - (\sqrt{25} + 3)]$
$= -8 - 12 \div [2 - (5 + 3)]$ (Power and roots first)
$= -8 - 12 \div [2 - 8]$ (Brackets)
$= -8 - 12 \div (-6)$
$= -8 - (-2)$ (Division)
$= -8 + 2$
$= -6$ (Addition)

G) Fractions

Always change to improper fractions (not mixed number) before doing any arithmetic on fractions
To **Add/Subtract fractions**, always change to **same denominator** first.
Example:

$6\frac{2}{5} - 2\frac{7}{10} = \frac{32}{5} - \frac{27}{10}$ (Change to improper fractions)
$= \frac{64}{10} - \frac{27}{10}$ (Change to same denominator)
$= \frac{37}{10}$ (Subtract numerator)
$= 3\frac{7}{10}$ (Change back to mixed number)

To **Multiply/Divide fractions**, **Don't change to same denominator**
Simply **multiply top by top** and **bottom by bottom**.
Example:

$2\frac{3}{4} \times \frac{2}{11} \times 4 \div 6$ (Change to improper fractions)
$= \frac{11}{4} \times \frac{2}{11} \times \frac{4}{1} \times \frac{1}{6}$ (Change divide to times and invert fraction)
$= \frac{\cancel{11}}{4} \times \frac{2}{\cancel{11}} \times \frac{4}{1} \times \frac{1}{6}$ (Cancel common factors from top and btm)
$= \frac{2}{6}$
$= \frac{1}{3}$ (change to simplest form)

H) Negative Numbers (Multiply/divide)

$Positive \times Positive = Positive$
$Negative \times Negative = Positive$
$Positive \times Negative = Negative$

*Division follows the same rules.
Examples
$2 \times 5 = 10$ $-4 \times (-10) = 40$ $-3 \times 15 = -15$
$-2(-100) = 200$ $-10 \div 2 = -5$ $-20 \div -4 = 5$

I) Real-Life Problems Involving Fractions Example

a) Find, in the simplest form, the fraction which is exactly halfway between -0.5 and $-\frac{5}{9}$
b) John is given $\frac{3}{8}$ of a sum of money and Mary receives $\frac{2}{5}$ of the remainder. If the amount of money left is $\$21$, find the original amount of money.

a) Required Fraction $= \frac{-\frac{1}{2} + \left(-\frac{5}{9}\right)}{2}$
$= \frac{\frac{-9-10}{18}}{2}$
$= \frac{-\frac{19}{18}}{2}$
$= -\frac{19}{36}$

b) Mary received $= \left(1 - \frac{3}{8}\right) \times \frac{2}{5} = \frac{1}{4}$ of the sum of money
Amount of money left $= 1 - \frac{3}{8} - \frac{1}{4}$
$= \frac{3}{8}$
Original amount of money $= \$\left(\frac{21 \times 8}{3}\right) = \$56$

J) Listing in order

List the following in ascending order (*Try both questions before looking at answers!*)
a) $\frac{1}{3}, 0.33\dot{3}, 0.\dot{3}\dot{1}, 0.333, 0.33^2$
b) $-0.\dot{7}1\dot{4}, -\frac{5}{7}, -0.71\dot{4}, \sqrt[3]{-0.365}$

a) $0.33^2, 0.\dot{3}\dot{1}, 0.33\dot{3}, 0.333, \frac{1}{3}$
b) $-0.\dot{7}1\dot{4}, \sqrt[3]{-0.365}, \ -0.71\dot{4}, -\frac{5}{7}$
(*Note that the negative number with a bigger number has a smaller value, i.e. -4 is smaller than -3)

K) Temperature Question Example

The temperature at the top of a 4800m mountain was -18^oC while the temperature at sea level was 6^oC. Given that the temperature changed uniformly with height, find
a) the temperature at 1800m above sea level,
b) the height above sea level where the temperature is 0^oC.

a) Difference in temperature $= 6 - (-18) = 24^oC$
$4800m \rightarrow 24^oC$
$1m \rightarrow \frac{24}{4800^oC}$
$1800m \rightarrow \frac{24}{4800} \times 1800 = 9^oC$
Since temperature decrease as we go higher, temperature at 1800m
$= 6 - 9$
$= -3^oC$

b) Difference in temperature from sea level is 6^oC
$24^oC = 4800m$
$6^oC = 1200m$
The height is 1200m above sea level

Self Practice

1) Real Numbers Example

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State all the numbers that are:

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g) Perfect Squares

2) Negative Numbers (Add/Subtract) Example

Without using a calculator, find the value of each of the following:

a) $11 - 23$ **b)** $-13 - 15$ **c)** $8 + 7$
d) $-8 + 11$ **e)** $-4 - (-6)$ **f)** $-6 - (-15)$

3) BODMAS Example

Without using a calculator, evaluate the value of

$(-2)^3 - 12 \div [2 - (\sqrt{25} + 3)]$

4) Real-Life Problems Involving Fractions Example

a) Find, in the simplest form, the fraction which is exactly halfway between -0.5 **and** $-\frac{5}{9}$

b) John is given $\frac{3}{8}$ **of a sum of money and Mary receives** $\frac{2}{5}$ **of the remainder. If the amount of money left is** $\$21$**, find the original amount of money.**

5) Listing in order

List the following in ascending order (*Try both questions before looking at answers!*)

a) $\frac{1}{3}, 0.31\dot{3}, 0.3\dot{1}, 0.333, 0.33^2$

b) $-0.\dot{7}1\dot{4}, -\frac{5}{7}, -0.71\dot{4}, \sqrt[3]{-0.365}$

6) Temperature Question Example

The temperature at the top of a 4800m mountain was -18^o **while the temperature at sea level was** 6^oC**. Given that the temperature changed uniformly with height, find**

a) the temperature at 1800m above sea level,

b) the height above sea level where the temperature is 0^oC**.**