Sec 2 Math: Trigonometry

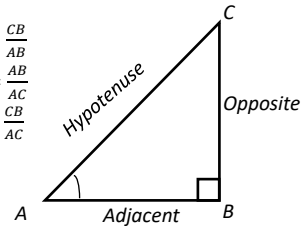
A) Hypotenus, Adjacent, Opposite

In the <u>right angled triangle</u> below, AC is the <u>Hypothenuse</u>, CB is the <u>Opposite</u> side and AB is the <u>Adjacent</u> side with respect to angle A

$\tan A = \frac{Opp}{Adj} = \frac{CB}{AB}$

$\cos A = \frac{Adj}{Hyp} = \frac{AB}{AC}$

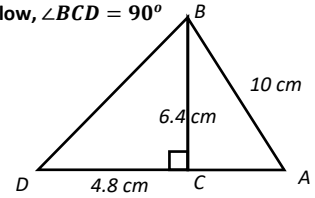
$\sin A = \frac{Opp}{Hyp} = \frac{CB}{AC}$



Use the Mnemonic ' TOA CAH SOH'

B) Finding Trigo Ratios

In the diagram below, $\angle BCD = 90^o$
Find
a) $\sin \angle CAB$
b) $\tan \angle CBD$
c) $\cos \angle CDB$



Note: The answers are fractions, not angles!

a) $\sin \angle CAB = \frac{Opp}{Hyp} = \frac{6.4}{10} = \frac{16}{25}$

b) $\tan \angle CBD = \frac{Opp}{Adj} = \frac{4.8}{6.4} = \frac{3}{4}$

c) $BC = \sqrt{6.4^2 + 4.8^2}$ (Pythagoras' Theorem)

$BC = 8$

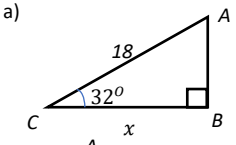
$\cos \angle CDB = \frac{6.4}{8}$

$\cos \angle CDB = \frac{4}{5}$

C) Finding Unknown Length

Find the value of x in the following triangles.

a)

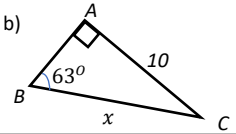


a) $\cos 32^o = \frac{x}{18}$

$18 \cos 32^o = x$

$x = 15.3$ (3 s.f.)

b)



b) $\sin 63^o = \frac{10}{x}$

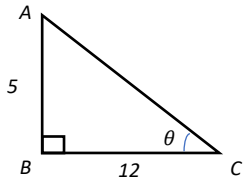
$x \sin 63^o = 10$

$x = \frac{10}{\sin 63^o}$

$x = 11.2$ (3 s.f.)

D) Finding Unknown Angle

In a right angled triangle, the side



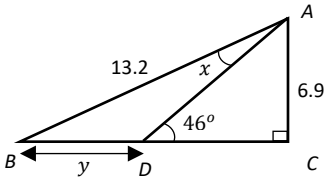
$\tan \theta = \frac{5}{12}$

$\theta = \tan^{-1}\left(\frac{5}{12}\right)$

$\theta = 22.6^o$

E) Mixed Trigo Question (Intermediate)

Find the unknowns in the following figures.



$\tan 46^o = \frac{6.9}{DC}$

$DC = \frac{6.9}{\tan 46^o}$

$DC = 6.6633$

$BC = \sqrt{13.2^2 - 6.9^2}$ (Pythag)

$BC = 11.253$

$y = 11.253 - 6.6633$

$y = 4.59$

Note: Pythagoras Theorem is often used together with trigo questions since they both apply to right angle triangles.

$\cos \angle CAB = \frac{6.9}{13.2}$

$\angle CAB = \cos^{-1}\left(\frac{6.9}{13.2}\right)$

$\angle CAB = 58.485^o$

$\angle DAC = 180^o - 46^o - 90^o$

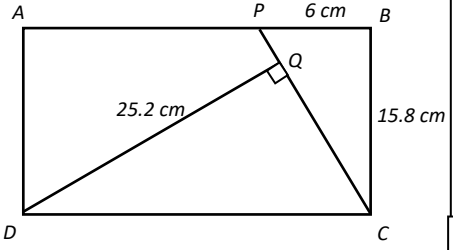
$\angle DAC = 44^o$

$x = 58.485 - 44$

$x = 14.5^o$

F) Trigonometry Question (Intermediate)

In the diagram, $ABCD$ is a rectangle, and DQ is perpendicular to CP. Given that $BC = 15.8$ cm, $PB = 6$cm and $DQ = 25.2$ cm, calculate,
a) $\tan \angle PCB$
b) $\angle DCQ$
c) AP
d) the area of ΔDQC



a) $\tan \angle PCB = \frac{Opp}{Adj}$

$= \frac{6}{15.8}$

$= \frac{30}{79}$

b) $\angle PCB = \tan^{-1}\frac{30}{79}$

$= 20.8^o$ (1 d.p.)

$\angle DCQ = 90^o - 20.8^o$

$= 69.2^o$

c) $\sin DCQ = \frac{25.2}{DC}$

$DC \sin(69.2059) = 25.2$

$DC = \frac{25.2}{\sin(69.2059)}$

$DC \approx 26.956$ cm

$DC = 27.0$ cm (3 s.f.)

$AP = 27.0 - 6$

$AP = 21$ cm

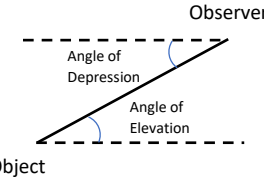
d) $QC = \sqrt{DC^2 - DQ^2}$ (Pythagoras)

$QC = 9.57$ cm

$Area\ of\ \Delta DQC = \frac{1}{2} \times 9.57 \times 25.2$

$= 121 cm^2$

G) Angle of Elevation and Depression

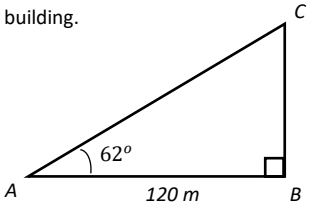


The angle of depression/elevation is the angle formed between the horizontal and the line from the observer's eye to the object.

The angle of depression is equal to the angle of elevation (as they are alternate angles).

H) Angle of Elevation Question

The angle of elevation of the top of a building at point A (on level ground) is 62^o. Point A is $120m$ away from the base of the building. Find the height of the building.



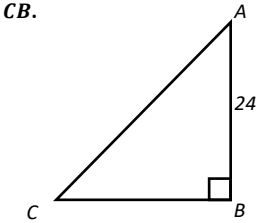
$\tan 62^o = \frac{h}{120}$

$h = 120 \times \tan 62^o$

$h = 226$ m (3 s.f.)

I) Using Trigo Ratios

In the diagram, $AB = 24$ cm and $\sin \angle ACB = \frac{3}{5}$, find the value of CB.



$\sin \angle ACB = \frac{24}{AC}$

$\frac{3}{5} = \frac{24}{AC}$

$3AC = 5 \times 24$

$AC = 40$

$CB = \sqrt{40^2 - 24^2}$ (By Pythagoras' Theorem)

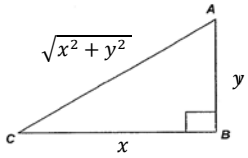
$CB = 32$

J) Drawing your own Triangle

In triangle ABC, $\angle ABC = 90^o$ and $\tan \angle BAC = \frac{x}{y}$.

Find, in terms of x and y, the expressions of
i) $\cos \angle ACB$
ii) $\sin(90^o - \angle BAC)$
**Tips: Draw your own right angle triangle*

$AC = \sqrt{x^2 + y^2}$
(Pythag's Thrm)

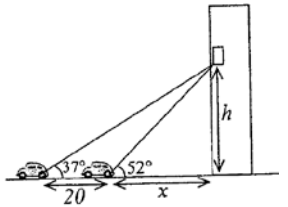


i) $\cos \angle ACB = \frac{x}{\sqrt{x^2+y^2}}$

ii) $\sin(90^o - \angle BAC)$
$= \sin(\angle ACB)$
$= \frac{y}{\sqrt{x^2+y^2}}$

K) Trigonometry Question (Advanced*)

Given the diagram below, find the value of x.



$\tan 52^o = \frac{h}{x}$

$x \tan 52^o = h$ ---(1)

$\tan 37^o = \frac{h}{20+x}$

$(20 + x) \tan 37^o = h$ ---(2)

Sub (1) into (2):

$x \tan 52^o = 20 \tan 37^o + x \tan 37^o$

$x(\tan 52^o - \tan 37^o) = 20 \tan 37^o$

$x = \frac{20 \tan 37^o}{\tan 52^o - \tan 37^o} = 28.6\ (3\ s.f.)$

Self Practice

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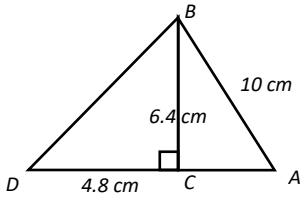

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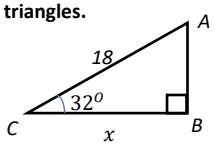

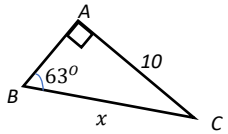

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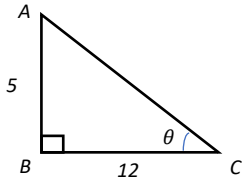

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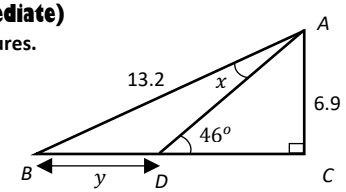

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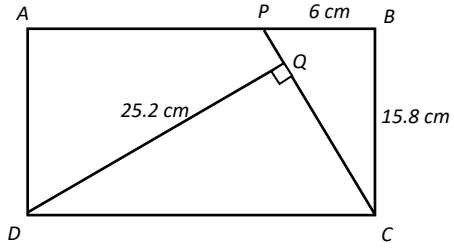

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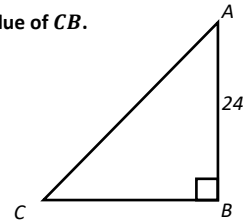

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